Statistics

Rijoy Kodiyan Jacob

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Rijoy K.J.

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- It has a long tail on one side and a short tail on the other side.
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Example

Income, Savings, etc. have skewed distributions.

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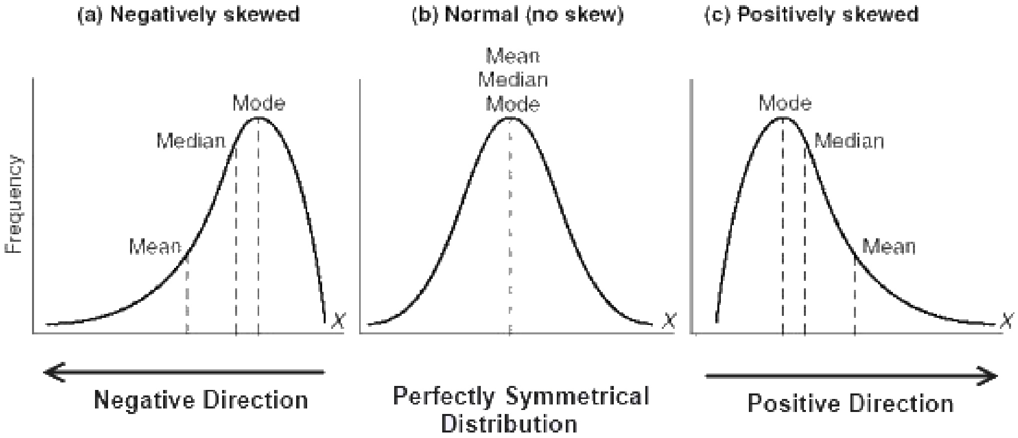
+ve and -ve Skewness



- Skewness may be either positive or negative.
- Skewness is said to be positive when mean is greater than median and median is greater than mode. In this case the curve is skewed to the right.
- Here more than half the area falls at the right side of the highest ordinate.
- Skewness is said to be negative when mean is less than median and median is less than mode (the curve is skewed to the left).
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Ist Measure of Skewness

Karl Pearson's Coefft.

$$\text{Karl Pearson's Coefficient 'J'} = \frac{Mean - Mode}{Standard\ \ Deviation}$$

In case, if in a frequency distribution, the Mode is ill defined, then Mean-Mode is taken as 3(Mean-Median)

If Mode Ill Defined

$$\text{'J'} = \frac{3(Mean - Median)}{StandardDeviation}$$; value of 'J' will be -3 $<$ J $>$ 3

2nd Measure of Skewness (Bowley's Coefft.)

$$\text{Bowley's coefft. 'J'} = \frac{(Q_3 + Q_1 - 2M)}{Q_3 - Q_1}$$

Where 'M' is the Median.

Third Measure of Skewness (Kelly's Coefft.)

$$\text{'J'} = \frac{(D_9 + D_1 - 2\ Median)}{(D_9 - D_1)} \text{ or } \frac{(P_{90} + P_{10} - 2\ Median)}{(P_{90} - P_{10})}$$

Where 'D' stands for decile and 'P'stands for Percentile.

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Fourth Measure of Skewness

On the basis of central moments, Coefficient of skewness is given by 'J' $= \dfrac{\mu_3}{\sqrt{\mu_2^2}}$

Where' μ_3 and μ_2 are third moment and second moment respectively.

Central Moment μ_r

Central Moment 'μ_r' $= \dfrac{\sum(x-\bar{x})^r}{n}$ for individual series and $\dfrac{\sum f \times (x-\bar{x})^r}{N}$ for a frequency distribution.

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Kurtosis

The term 'Kurtosis' indicates whether a distribution is flat topped or peaked.

Measure of Kurtosis is therefore measure of peakedness.

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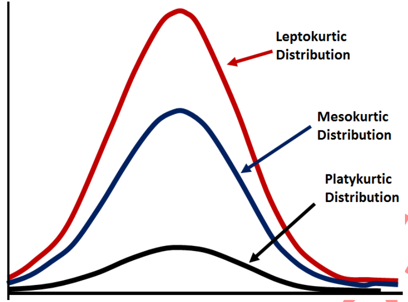
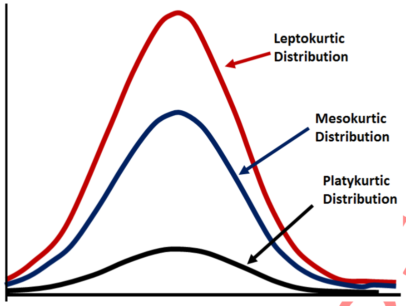


Leptokurtic

When a frequency curve is **more peaked** than the normal curve it is called **leptokurtic**.

Platykurtic

When a frequency curve is **more flat topped** than the normal curve, it is called **platykurtic**.



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Measure of Kurtosis

Measure of Kurtosis is derived from moments.

Measure of Kurtosis 'β_2' $= \dfrac{\mu_4}{(\mu_2)^2}$

When $\beta_2 = 3$, the distribution will be mesokurtic, if it is lower than 3, the distribution is platykurtic and if it is greater than 3, it is leptokurtic.

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Central Moments

Table: The first four moments are as follows

Individual Series	Frequency Distribution
$\mu_1 = \frac{\sum (x-\bar{x})^1}{n}$	$\mu_1 = \frac{\sum f \times (x-\bar{x})^1}{N}$
$\mu_2 = \frac{\sum (x-\bar{x})^2}{n}$	$\mu_2 = \frac{\sum f \times (x-\bar{x})^2}{N}$
$\mu_3 = \frac{\sum (x-\bar{x})^3}{n}$	$\mu_3 = \frac{\sum f \times (x-\bar{x})^3}{N}$
$\mu_4 = \frac{\sum (x-\bar{x})^4}{n}$	$\mu_4 = \frac{\sum f \times (x-\bar{x})^4}{N}$

Example

Find the coefficient of Skewness and Measure of Kurtosis for the following frequency distribution.

Class	0-2	2-4	4-6	6-8	8-10
Frequency	2	3	3	1	1

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Solution

Class	x	f	fx	$x - \bar{x}$	f$(x - \bar{x})$	$f(x - \bar{x})^2$	$f(x - \bar{x})^3$	$f(x - \bar{x})^4$
0-2	1	2	2	-3.2	-6.4	20.48	-65.54	209.73
2-4	3	3	9	-1.2	-3.6	4.32	-5.18	6.22
4-6	5	3	15	0.8	2.4	1.92	1.54	1.23
6-8	7	1	7	2.8	2.8	7.84	21.95	61.46
8-10	9	1	9	4.8	4.8	23.04	110.59	530.83
		10	42	4	0	57.60	63.36	809.47

$$\bar{x} = \frac{f \times \sum x}{N} = \frac{42}{10} = 4.2$$

$$`\mu_1' = \frac{\sum f \times (x - \bar{x})^1}{N} = \frac{0}{N} = 0$$

$$`\mu_2' = \frac{\sum f \times (x - \bar{x})^2}{N} = \frac{57.60}{10} = 5.76$$

$$`\mu_3' = \frac{\sum f \times (x - \bar{x})^3}{N} = \frac{63.36}{10} = 6.34$$

$$`\mu_4' = \frac{\sum f \times (x - \bar{x})^4}{N} = \frac{809.47}{10} = 80.95$$

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$$\text{Coefficient of skewness 'J'} = \frac{\mu_3}{\sqrt{(\mu_2)^3}} = \frac{6.36}{\sqrt{(5.76)^3}} = 0.46$$

$\therefore$ the distribution is positively skewed.

$$\text{Measure of Kurtosis} = \text{'}\beta_2\text{'} = \frac{\mu_4}{(\mu_2)^2} = \frac{80.95}{(5.76)^2} = 2.44$$

$\therefore$ the distribution is platykurtic.

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