Statistics

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Quartile Deviation(Semi Inter Quartile Range)

Quartile Deviation is defined as half the distance between the third and first quartiles.

Quartile Deviation is am absolute measure and it is given by

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

Q_1 and Q_3 are the first quartile and third quartile respectively.

The corresponding relative measure is the Coefficient of Quartile Deviation which is given by

$$\text{Coefficient of Quartile Deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

QD in Individual Series

$Q_1 =$ size of $\left(\frac{n+1}{4}\right)^{th}$ item, when the items are arranged in ascending order.

$Q_3 =$ size of $\left(\frac{n+1}{4} \times 3\right)^{th}$ item, when the items are arranged in ascending order.

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of Quartile Deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$\text{Inter Quartile Range} = Q_3 - Q_1$$

Example

Compute Quartile measure of dispersion, Interquartile range, and Coefficient of Quartile Deviation for the following series.
23, 25, 8, 10, 9, 29, 45, 85, 10, 16, 24.

Solution

If values are arranged in the ascending series,
8, 9, 10, 10, 16, 23, 24, 25, 29, 45, 85.; $N = 11$

$$Q_1 = \text{size of } \left(\frac{11+1}{4}\right)^{th} \text{ item} = \text{size of } 3^{rd} \text{ item} = 10$$

$$Q_3 = \text{size of } \left(\frac{11+1}{4} \times 3\right)^{th} \text{ item} = \text{size of } 9^{th} \text{ item} = 29$$

Q. D.

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{29-10}{2} = 9.5$$

Interquartile Range

Inter Quartile Range = Q_3 - Q_1 = 19

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{29-10}{29+10} = \frac{19}{39} = 0.49$$

QD in Discrete Series

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2},$$

$$Q_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ item, } N = \sum f$$

$$Q_3 = \text{size of } \left(\frac{3N+1}{4}\right)^{th} \text{ item, } N = \sum f$$

Example

Find the QD for the following series

Size	5	8	10	12	19	20	32
Freq.	3	10	15	20	8	7	6

Solution

Size	Freq	Cum. Freq
5	3	3
8	10	13
10	15	28
12	20	48
19	8	56
20	7	63
32	6	69
	$\sum N=69$	

$$Q_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$= 17.5 \text{ item size} = 10$$

$$Q_3 = \text{size of } \left(\frac{3N+1}{4}\right)^{th} \text{ item}$$

$$=52.5 \text{ item size} = 19$$

$$QD = \frac{Q_3 - Q_1}{2} = \frac{19-10}{2} = 4.5$$

QD in a Continuous Series

Here QD is the the same, i.e. $\text{QD} = \dfrac{Q_3 - Q_1}{2}$

Q_1 & Q_3 are size of $\left(\dfrac{N}{4}\right)$th item and size of $\left(\dfrac{N}{4} \times 3\right)$th item respectively. They cannot be determined directly and is determined graphically by the interpolation formula which is given by.

$$\text{Q}_1 = l_1 + \frac{\left(\dfrac{N}{4} - cf\right)}{f} \times c \quad \text{and} \quad \text{Q}_3 = l_1 + \frac{\left(\dfrac{3N}{4} - cf\right)}{f} \times c$$

Where
'l_1' is the lower limit of the quartile class.

'f' is the frequency of the class.

'cf' is the cumulative frequency of the preceding class.

'c' is the class interval of the quartile class.

Example

Find the quartile deviation for the following frequency distribution.

Age	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No.	15	30	53	75	100	110	115	125

Solution

First you have to form the cumulative frequency distribution and determine Q_1 and Q_3

$$Q_1 = l_1 + \frac{\left(\frac{N}{4} - cf\right)}{f} \times c \qquad Q_3 = l_1 + \frac{\left(\frac{3N}{4} - cf\right)}{f} \times c$$

Size	f	cf	
0-10	15	15	
10-20	30	45	
20-30	53	98	
30-40	75	173	$\frac{N}{4}$
40-50	100	273	
50-60	110	383	
60-70	115	498	$\frac{3N}{4}$
70-80	125	623	
	$\sum = 623$		

$\frac{N}{4}$ item size $= \frac{623}{4}$ th item $=$ 155.75

Lower quartile class is 30 - 40

$$Q_1 = l_1 + \frac{\left(\frac{N}{4} - cf\right)}{f} \times c = 30$$

$$+ \left(\frac{(155.75 - 98)}{75}\right) \times 10 = 37.7$$

$\frac{3N}{4}$ item size $= \frac{3 \times 623}{4}$ th item $= 467.5$

Lower quartile class is 60 - 70

$Q_3 = l_1 + \frac{\left(\frac{3N}{4} - cf\right)}{f} \times c = 60 + \left(\frac{(467.5 - 383)}{115}\right) \times 10 =$ 67.33

Quartile Deviation $= \frac{Q_3 - Q_1}{2} = \frac{67.33 - 37.7}{2} = 14.815$

Mean Deviation

Mean Deviation is defined as the arithmetic mean of deviations of all the values in a series from their average, counting all such deviations as positive. The average selected may be mean, median or mode.

$$\therefore \text{Mean Deviation} = \frac{\sum |d|}{n}$$

where '$|d|$' represents deviation from an average without sign. 'n' being number of items.

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If Mean is the chosen average,

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Example

Find the mean deviation from mean and its coefficient for the following values 25,63,85,75,62,70,83,28,30,12

Solution

First you have to find out the Mean.
Here the mean is obtained as

$$\text{Mean} = \frac{25 + 63 + 85 + 75 + 62 + 70 + 83 + 28 + 30 + 12}{10} = 53.3$$

Then find out deviations '$|\ d\ |$ for each value i.e. 25-53.3 = 28.3

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x	|d|= x - 53.3 without sign
25	283
63	9.7
85	31.7
75	21.7
62	8.7
70	16.7
83	29.7
28	25.3
30	23.3
12	41.3
533	236.4

$$\text{Mean Deviation} = \frac{\sum |d|}{n} = \frac{236.4}{10} = 23.64$$

$$\text{Coefficient of mean deviation} = \frac{Mean \quad Deviation}{Mean} = \frac{23.64}{53.3} = 0.44$$

Mean Deviation from Median

Calculate mean deviation from median and its coefficient for the following values, 5, 25, 28, 33,35, 44, 82, 83, 87, 96, 99

Solution

Median $=$ Size of $\dfrac{(N+1)}{2}$ th item.$=$ Size of 6th item $=44$.

Find out the deviation of each value from median. Find the sum of deviation.

Mean Deviation from Median

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Solution

Median $=$ Size of $\dfrac{(N+1)}{2}$ th item.$=$ Size of 6th item $= 44$.

Find out the deviation of each value from median. Find the sum of deviation.

x	|d|=x - 44 without sign
5	39
25	19
28	16
33	11
35	9
44	0
82	38
83	39
87	43
96	52
99	55
	321

$$\text{Mean Deviation} = \frac{321}{11} = 29.18.$$

$$\text{Coefficient of Mean Deviation} = \frac{29.18}{44} = 0.66$$

Mean Deviation in Discrete Frequency Series

In the case of discrete frequency series, MD is given by,

$$\text{MD} = \frac{\sum f \times \mid d \mid}{N}$$

Where 'f' is the frequency corresponding to the given size, '$\mid x \mid$' and 'N' is the total frequency.

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Example

Calculate the MD for the following series from mean.

Size	0	1	2	3	4	5	6
Frequency	171	82	50	25	13	7	2

x	f	fx	|d| =|x – 1.02|	f|d|
0	171	0	1.02	174.42
1	82	82	0.02	1.64
2	50	100	0.98	49.00
3	25	75	1.98	49.5
4	13	52	2.98	38.74
5	7	35	3.98	27.86
6	2	12	4.98	9.96
	350	356		351.12

$$\text{Mean of the series} = \frac{\sum f \times x}{N} = \frac{356}{350} = 1.02$$

$$\text{Mean Deviation} = \frac{\sum f \times \mid d \mid}{N} = \frac{351.52}{350} = 0.98$$

MD in Continuous Frequency Distribution

MD in continuous frequency distribution is given by:

$$\text{Mean Deviation} = \frac{\sum f \times \mid d \mid}{N}$$

It is same as discrete frequency series. The only difference is that here the mid value of the class is taken as 'x'.

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Example

Calculate the MD from mean for the following frequency distribution.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	6	10	20	10	6	4

Marks	f	Mid x	fx	|d| =|x – 35|	f|d|
0-10	4	5	20	30	120
10-20	6	15	90	20	120
20-30	10	(25)	250	10	100
30-40	20	35	700	0	0
40-50	10	45	450	10	100
50-60	6	55	330	20	120
60-70	4	65	260	30	120
	60		2100		680

Solution

$$\text{Mean of the series} = \frac{\sum f \times x}{N} = \frac{2100}{60} = 35$$

$$\text{Mean Deviation} = \frac{\sum f \times \mid d \mid}{N} = \frac{680}{60} = 11.33$$

Merits

- Mean deviation is a very simple and an easy measure of dispersion.
- It is easily understood.
- It is based on all the items of the series. So it is more representative.
- Mean deviation is less affected by extreme values.

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- Mean deviation suffers from inaccuracy because '$+$' or '$-$' signs are ignored.
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- Mean deviation is significantly used (or measuring variability of the series relating to Economic and Social phenomena.
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