MODULE – III

Aims and Objectives

This Lesson deals with the concepts and applications of sequence and series. Applications of series like Arithmetic Progression and Geometric Progression and practical applications

Sequence

If for every positive integer n, there corresponds a number a_n such that a_n is related to n by some rule, then the terms a_1, a_2,….a_n…. are said to form a sequence.
A sequence is denoted by bracketing its n^{th} term, i.e. (a_n) or {a_n}.
Example of a few sequences are:

1. If $a_n = n^2$, then sequence {a_n}is 1,4,9,16….a_n,…
2. If $a_n = 1/n$, then sequence {a_n} is 1,1/2,1/3,1/4…1/n…
3. If $a_n = n^2/n+1$, then sequence {a_n}is ½, 4/3, 9/4,…$n^2/n+1$,….

The concept of sequence is very useful in finance. Some of the major areas where it plays a vital role are: "instalment buying'; simple and compound interest problems'; 'annuities and their present values', mortgage payments and so on

Series

A series is formed by connecting the terms of a sequences with plus or minus sign. Thus if an is the nth term of a sequence, then
$a_1 + a_2 + \dots + a_n$ is the given series of n terms.

Arithmetic Progression (AP)

A progression is a sequence whose successive terms indicate the growth or progress of some characteristics. An arithmetic progression is a sequence whose term increases or decreases by a constant number called common difference of an A.P. and is denoted by d. In other words, each term of the arithmetic progression after the fist is obtained by adding a constant d to the preceding term. The standard form of an A.P. is written as

a, a+d, a+2d, a+3d,…

where 'a' is called the first term. Thus the corresponding standard form of an arithmetic series becomes

a+(a+d)+(a+2d)+(a+3d)+….

For example

1.The sequence 1, 3, 5, 7, ... is an A.P whose first term is 1 and d = 2
2.The sequence -5, -2, 1, 4, 7, ... , whose 'a' = -5, d = 3

Suppose we invest Rs. 100 at a simple interest of 15% per annum for 5 years. The amount at the end of each year is given by

115,130,145,160,175

This forms an arithmetic progression

The n^th Term of an A.P.

The n^th term of an A.P. is also called the general term of the standard A.P. it is given by.

$$T_n = a+(n-1)d; \quad n=1,2,3,\ldots$$

Geometric Progression (GP)

A geometric progression (GP) is a sequence whose each term increases or decreases by a constant ratio called common ratio of G.P. and is denoted by r. In other words, each term of

G.P. is obtained after the first by multiplying the preceding term by a constant r. The standard from of a G.P. is written as :

$a, ar, ar^2, \ldots.$

Where 'a' is called the first term. Thus the corresponding geometric series in standard form becomes

$a + ar + ar^2 + \ldots.$

The n^th Term of G.P.

The n^th term of G.P. is also called the general term of the standard G.P. It is given by $T_n=ar^{n-1}$, n=1,2,3,…

It may be noted here that the power of r is one less than the index of T_n, which denotes the rank of this term in the progression.

Practical Problems

1) Find the 12^th term of an A.P 6, 2, -2

Ans: $a_n = a + (n-1)d$

a = 6, n = 12, d = -4

= 6 + (12-1) – 4

= 6 + (11) – 4

= 6 + ⁻44 = -38

12^th term is -38

2) Find the 8^th term of the series 6, 5½, 5, 4½, ...

Ans: a = 6, d = -½,n = 8

$a_n = a + (n-1)d$

$= 6 + (8-1)-½$

$= 6 + (7) -½$

$= 6 + -3.5 = 2.5$

3) Which term of the A.P 21, 18, 15, ... -81 ?

Ans: $a = 21$, $d = -3$, $a_n = -81$ $n = ?$

$a_n = a + (n-1)d$

$-81 = 21 + (n-1)-3$

$-81 = 21 + -3n +3$

$-81 = 24 – 3n$

$-81 – 24 = -3n$

$3n = 105$

$n = 105/3 = 35$

Therefore the 35^{th} term of the given A.P = -81

4) Which term of the A.P 21,18,15, ... 0 ?

Ans: $a = 21$, $d = -3$, $a_n = 0$, $n = ?$

$a_n = a + (n-1)d$

$0 = 21 + (n – 1)-3$

$0 = 21 + -3n + 3$

$0 = 24 – 3n$

$3n = 24$, $n = 8$

Therefore, the 8^{th} term = 0

5) If the 9^{th} term of an A.P is 99 and 99^{th} term is 9. Fine 108^{th} term?

Ans: $a_n = a + (n-1)d$

$n = 9$, $a_n = 99$

$= a + (9-1)d = 99$

$= a + 8d = 99$ ----------------(1)

$n = 99$, $a_n = 9$

$= a + (99 – 1)d =9$

$= a + 98d = 9$ ---------------(2)

Solve the equations

$a + 8d = 99$ ----------------(1)
$a + 98d = 9$ ----------------(2)

Then (1) – (2) $-90d = 90$

$d = 90/-90 = -1$

Substitute the value of 'd'

$a + 8d = 99$

$a + 8 \times -1 = 99$

$a + -8 = 99$

$a = 99 + 8 = 107$

108^{th} term = $a + (n-1)d$

$= 107 + (108 - 1)-1$

$= 107 + (107)-1$

$= 107 - 107 = 0$

108^{th} term = 0

.6) Determine the A.P whose 3^{rd} term is 5 and the 6^{th} term is 8

Ans: $a + 2d = 5$ -------------------- (1)
$a + 5d = 8$ --------------------(2)

Then (1) – (2) = $-3d = -3$

$$d = \frac{3}{3} = 1$$

A.P = 3, 4, 5, 6, 7, 8 ……………….

7) Find many two digit numbers are divisible by 3 ?

Ans: Numbers = 12, 15, 18, - - - - - - 99

$a = 12$, $d = 3$, $a_n = 99$

$a_n = a + (n-1)d$

$99 = 12 + (n-1)3$

$99 = 12 + 3n - 3$

$99 = 12 - 3 + 3n$

$99 = 9 + 3n$

$3n = 99 - 9$, $3n = 90$

$$n = \frac{90}{3} = 30$$

∴ Two digit numbers are divisible by 3 = 30 number

8) Determine the 25th term of the A.P, whose 9th term is -6 and the common difference is 5/4.

Ans: d = 5/4, a_9 = -6

$a_9 = a + (n-1)d$

$-6 = a + 8 \times \frac{5}{4}$

$-6 = a + 10$

$a = -10 - 6 = -16$

$a_{25} = a + (n - 1)d$

$= -16 + (25 - 1)\ \frac{5}{4}$

$= -16 + 24 \times \frac{5}{4}$

= -16 + 30 = 14

25th term = 14

Sum of n terms of an A.P

Let S_n denotes the sum of '*n*' terms of an A.P, whose first term is '*a*' and common difference is '*d*'.

$$S_n = {}^{n}/_{2}\,[2a + (n-1)d]$$

$2a = a + a$ or $2 \times a$

Practical Problems

(1) Find the sum of the first 20 terms of 1 + 4 + 7 + 10 ....

Ans: $S_n = {}^{n}/_{2}\,[2a + (n-1)d]$

n = 20, a = 1, d = 3

$S_n = \frac{20}{2}(2 \times 1 + (20 - 1)3$

= 10 (2+ 19 x 3)

= 10(2 + 57), 10 x 59 =590

Sum of the first 20 terms = 590

2) Find the sum of the series 5, 3, 1, -1, ... -23

Ans: a= 5, d = -2, n = ?, a_n =-23

$S_n = {}^{n}/_{2}\,[2a + (n-1)d]$

We know, $a_n = a + (n - 1)d$

-23 = 5 + (n – 1)-2

-23 = 5 + -2n + 2

-23 = 5 + 2 – 2n

-23 = 7 – 2n

2n = -23 – 7

2n = 30, n = $\frac{30}{2}$ = 15

$S_n = \frac{15}{2}$ (2 x 5 + (15 – 1)-2)

= $\frac{15}{2}$ (10 + 14 x-2)

= $\frac{15}{2}$ (10 + -28)

= $\frac{15}{2}$ x -18 = 15 x -9 = -135

Sum of the series = -135

3) How many terms of the sequence 54, 51, 48, ... be taken so that their sum is 513. Explain the double answer.

Ans: S_n = 513, a = 54, d = -3

$S_n = {}^{n}/_{2}\,[2a + (n-1)d]$

513 = $\frac{n}{2}$ (2 x 54 + (n – 1)-3)

513 = $\frac{n}{2}$ (108 – 3n + 3)

513 = $\frac{n}{2}$ (111 – 3n)

= 1026 = n(111 – 3n)

= 1026 = 111n – $3n^2$

= $3n^2$ – 111n = -1026

= $3n^2$ – 111n + 1026 = 0

$= n^2 - 37n + 342 = 0$

Solve by using quadratic formula

i.e., $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1, \quad b = -37, \quad c = 342$

$n = \frac{37 \pm \sqrt{37^2 - 4x1x342}}{2x1}$

$= \frac{37 \pm \sqrt{1369 - 1368}}{2}$

$= \frac{37 \pm \sqrt{1}}{2} = \frac{37 \pm 1}{2}$

$= \frac{37+1}{2}$ or $\frac{37-1}{2}$

= 19 or 18

N = 18 or 19

4) Find the sum of all natural numbers between 500 and 1000 which are divisible by 13.

Ans: Number between 500 and 1000 which are divisible by 13

507, 520, 533, ... 988

$a = 507, \quad d = 13, \quad a_n = 988$

$a_n = a + (n-1)d$

988 = 507 + (n-1)13

988 = 507 + 13n – 13

988 = 507 – 13 + 13n

988 = 494 + 13n

13n = 988 – 494 = 494

13n = 494

$n = \frac{494}{132} = 38$

$S_n = {}^{n}/_{2}\,[2a + (n-1)d]$

= 19(1014 +37 x 13)

= 19(1014 + 481)

= 19 1495 = 28405

5) Find the sum of all natural numbers from 1 to 200 excluding those divisible by 5

Ans: Natural number from 1 to 200 = 1, 2, 3, 4, ... 200

Divisible by 5 = 5, 10, 15, 20 200

∴ Natural numbers from 1 to 200, excluding divisible by 5 =

(1, 2, 3, 4 . . . . 200) – (5, 10, 15 . . . . . 200)

Sum of (1, 2, 3, 4, ... 200) =

$S_n = {}^{n}/_{2}\,[2a + (n-1)d]$

$= \frac{200}{2}\left[2\text{x}1 + (200-1)1\right]$

= 100 (2+199)

=100 x 201 = 20,100

Sum of (5, 10, 15,20, ... 200)

$= \frac{40}{2}(2\text{ x}5 + (40-1)5)$

=20 (10 + 39 x 5)

=20(10 + 195)

= 20 x 205 = 4100

Sum by natural numbers from 1 to 200 excluding divisible by 5 = 20100 – 4100

= 16000

6) The sum of the first 3 terms of an A.P is 30 and the sum of first 7 terms is 140. Find the sum of the first 10 terms.

Ans: $S_3 = 30$, $s_7 = 30$,

$S_n = {}^{n}/_{2}\,[2a + (n-1)d]$

$= \frac{3}{2}\left[2a + (3-1)d\right] = 30$

$= 2a + 2d = 30 \text{ x} \frac{2}{3}$

= 2a + 2d = 20

= a + d = 10 --------------(1)

$= \frac{7}{2}\left[2a + 6d\right] = 140$

$= 2a+6d = 140 \times \frac{2}{7}$, $= 2a + 6d = 40$

a + 3d = 20 ---------------(2)

Solving the equation (1) and (2) d = 5

Then a = 5

$S_{10} = \frac{10}{2}\left[2 \times 5 + 9 \times 5\right] = \underline{\underline{275}}$

7) Find three numbers in A. P whose sum is 9 and the product is -165.

Ans: Let the numbers be a-d, a, a+d

(a-d) + a + (a +d) = 9

3a = 9, a = 3

$(a\text{-d}) \times a \times (a\text{+d}) = -165$

= (3 – d) x 3 x (3 +d) = -165

$= 9 - d^2 = \frac{-165}{3}$

$= 9 - d^2 = -55$

$= -d^2 = -55 - 9 = -64$

$= d^2 = 64$, d = 8

$a = 3$, d = 8

Numbers = (a – d), a, (a+d)

$= \underline{\underline{-5, 3, 11}}$

8) Find four numbers of A.P whose sum is 20 and the sum of whose square is 120

Ans: Let numbers be (a-3d), (a-d) (a+d) (a+3d)

Given (a-3d)+(a-d)+(a+d)+(a+3d) = 20

4a = 20, $a = \frac{20}{4} = 5$

$(a-3d)^2 \times (a-d)^2 \times (a+d)^2 \times (a+3d)^2 = 120$

$= (5-3d)^2 \times (5-d)^2 \times (5+d)^2 \times (5+3d)^2 = 120$

We know $(a-b)^2 = a^2 - 2ab + b^2$

$= 25 - 30d + 9d^2 + 25 - 10d + d^2 + 25 + 10d + d^2 + 25 + 30d + 9d^2 = 120$

$= 100 + 20d^2 = 120$

$20d^2 = 120 = 100$

$20d^2 = 20, \quad d^2 = 20/20 = 1, \quad d = 1$

$a = 5, \quad d = 1$

Numbers are = (a – 3d), (a-d), (a + d), (a + 3d)

= (5-3). (5-1), (5+1), (5+3)

= 2, 4, 6, 8

9) A manufacturing of radio sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the production uniformly increases by a fixed number every year. Find

(1) One production in the first year

(2) The production in the 10th year.

(3) The total production in 7 year.

Ans: Since the production increases uniformly by a fixed number in every year, it form an A.P.

Let $a_3 = 600, \quad a_7 = 700$

$a_n = a + (n - 1)d$

$600 = a + (3-1)d$

$600 = a + 2d$ .... (1)

$700 = a + 6d$ .... (2)

$a + 2d = 600$ ....(1)
$a + 6d = 700$ ....(2)

$-4d = -100$

$$d = \frac{100}{4} = 25$$

(1) Production in the first year

$a + 2d = 600$
$a + 50 = 600$
$a = 550$

(2) Production in the 10th year

i.e., $a_n = a + (n-1)d$
$= 550 + (10 - 1)\, 25$
$= 550 + 9 \times 25$
$= 550 + 225 = 775$

(3) Total production in 7th year

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{7}{2}\left[2 \times 550 + (7-1)25\right]$$

$$= \frac{7}{2}(1100 + 6 \times 25)$$

$$= \frac{7}{2}(1100 + 150)$$

$$= \frac{7}{2}(1250)$$

$$= 7 \times 625 = \underline{\underline{4375 \text{ units}}}$$

10) The rate of monthly salary of a person is increased annually in A.P. It is known that he was drawing as 400 a month during the 11th year of his service and as 760 during the 29th year. Find

(1) Starting salary

(2) Annual increment

(3) Salary after 36 years.

Ans: $a_{11} = 400$

$a_{29} = 760$

$$a + 10d = 400$$

$$a + 28d = 760$$

$$-18d = -360$$

$$d = \frac{360}{18} = 20$$

$$a + 10d = 400$$

$$a + 10 \times 20 = 400$$

$$a + 200 = 400$$

$$a = 400 - 200 = 200$$

$$a_{36} = 200 + 35d$$

$$200 + 35 \times 20$$

$$200 + 700 = \underline{\underline{900}}$$

1) Starting salary = 200
2) Annual Increment = 20
3) Salary after 36 years = 900

Arithmetic Mean (A.M)

Given two numbers a and b, we can insert a number A between them, so that a, A, b is an A.P. Such a number A is called the Arithmetic Mean of the number a and b.

We can insert as many numbers as we like between them. Let A, A_2, A_3 . . . . . A_n be 'n' numbers between a and b,

Then

$A_1 = a + d$

$A_2 = a + 2d$

$A_3 = a + 3d$

$A_n = a + nd$

Example

1) Find A.M between 2 and b

Ans: A.M between 2 and 6 = $\frac{2+6}{2} = 4$

Then A.P. = <u>2, 4, 6</u>

2) Insert 4 Arithmetic means between 5 and 20

a = 5, n = 6, a_n =20, d = ?

$a_n = a + (n - 1)d$

20 = 5 + (6-1)d

20 = 5 + 5d

20 = 5 + 5d

5d = 20 – 5 = 15

d = 15/5 = 3

$A_1 = a + d$ i.e., 5 +3 = 8

$A_2 = a + 2d$ i.e., 5 +6 = 11

$A_3 = a +3d$ i.e., 5 +9 = 14

$A_4 = a + 4d$ i.e., 5 +12 = 17

Arithmetic means are <u>8, 11, 14, 17</u>

A.P. = <u>5, 8, 11, 14, 17, 20</u>

3) Insert six numbers between 3 and 24 such that the resulting sequence is an A.P.

Ans: $a = 3$, $n = 8$, $a_n = 24$, $d = ?$

$a_n = a + (n - 1)d$

$24 = 3 + 7d$

$7d = 21$, $d = 3$

$A_1 = 3 + 3 = 6$

$A_2 = 3 + 6 = 9$

$A_3 = 3 + 9 = 12$

$A_4 = 3 + 12 = 15$

$A_5 = 3 + 15 = 18$

$A_6 = 3 + 18 = 21$

A.M. = 6, 9, 12, 15, 18, 21

A.P. = 3, 6, 9, 12, 15, 18, 21, 24

Geometric Progression

A series is said to be in G.P if every term of it is obtained by multiplying the previous term by a constant number. This constant number is called common ratio, denoted by 'r'. $r = \frac{second\ term}{first\ term}$ or third term by second term etc.

The first term of a G.P is usually denoted by a. The general form of a G.P is usually denoted by a. The general form of a G.P is a, ar, ar^2, ar^3 ..... If the number of terms of a G.P is finite, it is called a finite G.P, otherwise it is called an infinite G.P. For example.

(i) 1, ½, ¼, $^1/_8$ .... is a G.P, whose first term is 1 and r = ½

(ii) 3, -6, 12, -24 .... is a G.P whose a = 3 , r = -2

General term of a G.P or n^{th} term of a G.P

Let 'a' be the first term and 'r' be the common ratio of a G.P, then

$$a_n = ar^{n-1}$$

1) Find 10^{th} term of series 9, 6, 4.........

Ans: $a = 9$, $r = \frac{6}{9} = \frac{2}{3}$, $n = 10$

$a_n = ar^{n-1} = 9 \times \left(^2/_3\right)^{10-1}$

$= 9 \times \left(^2/_3\right)^9 = 9\left(^2/_3\right)^9$

2) Find the 12th term of 2, 6, 18, 54 ....

$a = 2,\quad r = 6/2 = 3,\quad n = 12$

$a_n = ar^{n-1} = 2 \times 3^{12-1}$

$= 2 \times 3^{11} = 2 \times 177147 = \underline{\underline{3,54,294}}$

3) Which term of the G.P 2, 8, 32 .... Up to n terms is 131072 ?

$a = 2,\quad r = 4,\quad a_n = 1{,}31{,}072$

$a_n = ar^{n-1}$

$1{,}31{,}072 = 2 \times 4^{n-1}$

$4^{n-1} = \frac{1{,}31{,}072}{2} = 65536$

$4^{n-1} = 65536$

i.e., $4^8 = 65536$

i.e. $n-1 = 8$

$\therefore n = 8 + 1 = 9$

<u>Hence 1,31,072 is the 9th term of the G.P.</u>

4) In a G.P the third term is 24 and 6th term is 192. Find the 10th term .

Ans: $a_3 = 24,\quad a_6 = 192$

$a_n = ar^{n-1}$

$a_3 = ar^2 = 24$

$a_6 = ar^5 = 192$

i.e., $ar^2 = 24$ --------- (1)

$ar^2 = 192$ -------(2)

Divide (2) by (1),

$\frac{ar^5}{ar^2} = \frac{192}{24}$

$r^3 = 8$ i.e., 2^3

$r = 2$

Substituting r = 2 in (1)

$ar^2 = 24,\quad a \times 2^2 = 24$

$a \times 4 = 24,\quad a = 24/4 = 6$

$a_{10} = ar^{n-1} = 6(2)^9 = \underline{\underline{3072}}$

Sum of 'n' terms of a G.P

Let 'a' be the first term and 'r' be the common ratio and S_n the sum of the 'n' terms of G.P.

Then $S_n = \frac{a(1-r^n)}{(1-r)}$ or $\frac{a(r^n-1)}{(r-1)}$

When r is less than 1, we can apply first formula.

1) Find the sum of the series.

1024 + 512 + 256 ....to 15 terms

Asn: a = 1024, n = 15, r = ½

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$= \frac{1024(1-½^{15})}{(1-½)}$$

$$= \frac{1024 \times (1/2)^{15}}{(1-½)}$$

$$= 1024 \quad \frac{2}{1} \times \left(\frac{1}{2}\right)^{15}$$

$$= 2048 \times \left(\frac{1}{2}\right)^{15}$$

2) Find the sum of 1 + 3+ 9 + 27 .... to 10 terms.

a = 1, r = 3, n = 10

$$S_n = \frac{a(r^n-1)}{(r-1)}$$

$$= \frac{1(3^{10}-1)}{(3-1)} = \frac{59049-1}{2} = 29524$$

3) How many terms of the G.P 3, 3/2, 3/4 , ... are needed to give the sum $\frac{3069}{512}$

Ans: a = 3, r = ½, $S_n = \frac{3069}{512}$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\frac{3069}{512} = \frac{3(1-(½)^n)}{(1-½)}$$

$= \frac{3069}{512} = \frac{3(1-(½)^n)}{½}$

$= \frac{3069}{512} = 3 \times \frac{2}{1}(1- \left(\frac{1}{2}\right)^n$

$= \frac{3069}{512} = 6\ (1- \left(\frac{1}{2}\right)^n$

$= \frac{3069}{512x6} = (1- \left(\frac{1}{2}\right)^n = \frac{3069}{3072} = 1- \frac{1}{2^n}$

$= \frac{1}{2^n} = 1- \frac{3069}{3072} = \frac{3}{3072} = \frac{1}{1024}$

$2^n = 1024$

$2^{10} = 1024,$ $n = 10$

4) Find three numbers in G.P whose sum is 14 and product is 64

Ans: Let the numbers = a/r, a, ar

$\therefore \frac{a}{r} + a + ar = 14$

$\frac{a}{r} \times a \times ar = 64$

$\therefore a^3 = 64$

$4^3 = 64$

$a = 4$

Substituting value of a

$\frac{a}{4} + a + ar = 14$

$\frac{4}{r} + 4 + 4r = 14$

Multiply by 'r'

Then $= 4 + 4r + 4r^2 = 14r$

$4r^2 - 10r + 4 = 0$

Use quadratic formula, for getting the value of 'r'

r = 2 cor ½

numbers = $\frac{a}{r}, a, ar$

r = 2 $= \frac{4}{2}, 4, 4 \times 2$, r = ½ = <u>8, 4, 2</u>

= <u>2, 4, 8</u>

Both are the same = <u>2, 4, 8</u>

5) A Person has 2 parents, 4 grand parents, 8 great grant parents and so on. Find the number of his ancestors during the ten generations preceding his won.

$a = 2$, $r = 2$, $n = 10$

$$s_n = \frac{a\,(r^n - 1)}{r-1}$$

$$s_{10} = \frac{2\,(2^{10} - 1)}{2-1}$$

$$= \frac{2\,(2^{10} - 1)}{1}$$

$$= 2\,(2^{10} - 1) = 2046$$

Number of ancestors preceding the person is 2046.

Geometric Mean

One geometric mean of two positive numbers a and b is the number $\sqrt{ab}$. Therefore, the geometric mean of 2 and 8 is 4. We can insert as many numbers as we like between a and b to make the sequence in a G.P. Let $G_1, G_2, G_3, \ldots\ldots\ldots\ldots G_n$ be 'n' number between a and b, then

$G_1 = ar$, $G_2 = ar^2$, $G_3 = ar^3$, $G_n = ar^n$

1) Insert three G.M. between 1 and 256

Ans. $a = 1$, $a_n = 256$, $n = 5$, $r = ?$

$$a_n = ar^{n-1}$$

$$256 = 1\,r^{n-1}$$

$$256 = r^{n-1}$$

$$256 = r^{5-1}$$

$$256 = r^4$$

$256 = 4^4$, $r = 4$

G.M. are ar, ar^2, ar^3

$1 \times 4, 1 \times 4^2, 1 \times 4^3 =$ 4, 16, 64

G.P = 1, 4, 16, 64, 256

2) Find the G.M between 4 is 16

Ans: G.M = $\sqrt{4 \times 16}$ = $\sqrt{64}$ = 8

3) Insert 5 geometric means between 2 and 1458

Ans: a = 2, n = 7, a_n = 1458

$a_n = ar^{n-1}$

$1458 = 2\,r^{7-1}$

$1458 = 2r^6$

$2r^6 = 1458$

$r^6 = \frac{1458}{2}$, $r^6 = 729$

$r^6 = 3^6$

∴ r = 3

G.M. = $ar, ar^2, ar^3, ar^4, ar^5$

= $2 \times 3, 2 \times 3^2, 2 \times 3^3, 2 \times 3^4, 2 \times 3^5$

= 6, 18, 54, 162, 486, 486

G.P. = 2, 6, 18, 54, 162, 486, 1458

4) If the A.M. between two positive numbers is 34 and their G.M. is 16. Find the numbers?

Ans: Let the numbers a and b

A.M. = $\frac{a+b}{2}$ = 34

G.M = $\sqrt{ab}$ = 16

∴ a +b = 68

$a \times b = 256$

$b = 68 - a$

$ab = 256$

$a(68-a) = 256$

$a^2 - 68a = 256$

$a^2 - 68a - 256 = 0$

Using quadratic formula

a = 4 or 64

When a = 4, b = 64

When $a = 64$, $b = 4$

Required numbers are 64 and 4

5) Find the three numbers in G.P whose sum is 26 and product is 216.

Ans: Let the number is G.P be

$a/r, a, ar$

$a/r, a/ ar = 216$

i.e. $a^3 = 216$, $6^3 = 216$

$\therefore a = 6$

$a/r + a + ar = 6/r + 6 + 6r = 26$

$= 6/r + 6r = 26 - 6$

$= 6/r + 6r = 20$

Multiply by r

$= 6 + 6^2 = 20r$

$= 6^2 - 20r + 6$

$= 6^2 - 20\,r + 6 = 0$

Solving by using quadratic formula

Then $r = 1/3$ or 3

Required numbers $a/r, a, ar$

$r = 3$

$6/3, 6, 6 \times 3 = 2, 6, 18$

Practical Problems

(1) Find the sum of the first 20 terms of 1 + 4 + 7 + 10 . . . .

(2) Find four numbers in A.P whose sum is 20 and the sum of squares are 120

(3)Find the 9^{th} term of the series 1,4,7................

(4) Find the n^{th} term of the series2,4,6,8...............

(5) if the third term of an AP is 3 and the 7^{th} term is 39. Find the common Difference

(6)Which term of the series is 17+23+29+..........is 551.

(7)7^{th} term and 12^{th} term of an A.P is 10 and 20. Find the first term.

(8) find the G.M between 4 and 16.

(9)The sum of first two terms of a GP is 2 and the sum of 4 GP is 20.Determine the GP

(10)Find five numbers in GP such that their product is 32 and the product of last two is 108.

(11). Find the 10th trm if 9,6,4...................................

Mathematics of finance

Aims and Objectives

So far we have discussed about various mathematical functions and theories. This Lesson deals with the applications of such theories in Finance. In financial management, lot of calculations are involved in the case of interest, depreciation values, and so on.

1. Some terms used in business calculations

Principal amount (P). This is the amount of money that is initially being considered. It might be an amount about to be invested or loaned or it may refer to the initial value or cost of plant or machinery. Thus if a company was considering a bank loan value or cost of plant or machinery. Thus if a company was considering a bank loan of say Rs.20000, this would be referred to as the principal amount to be borrowed.

Accrued amount (A). This term is applied generally to a principal amount after some time has elapsed for which interest has been calculated and added. It is quite common to qualify a precisely according to time elapsed. Thus A^1, A^2, etc would mean the amount accrued at the end of the first and second years and so on. The company referred to in (a) above might owe, say, an accrued amount of Rs.22000 at the end of the first year and Rs. 24200 at the end of the second year (if no repayments had been made prior to this time).

Rate of interest (i). Interest is the name given to a proportionate amount of money which is added to some principal amount (invested or borrowed). It is normally denoted by symbol i and expressed as a percentage rate per annum. For example if Rs. 100 is invested at interest rate 5% per annum (pa), it will accrue to Rs. 100 + (5% of Rs. 100) = Rs 100 + Rs.5 = Rs.105 at the end of one year. Note however, that for calculation purposes, a percentage rate is best written as a proportion. Thus, an interest rate of 10% would be written as i = 0.1 and 12.5% as i = 0.125 and so on.

Number of time periods (n). The number of time periods over which amounts of money are being invested or borrowed is normally denoted by the symbol n. although n is usually a number of years, it could represent other time periods, such as a number of quarters or months.

Simple interest

It is the interest calculated on principal amount at the fixed rate .

$$\text{Simple Interest} = \frac{Pnr}{100}$$

Where P = Principal amount, n = number of year,

r = rate of interest per annum

$$\text{Amount at the end of } n^{th} \text{ year} = P + \frac{Pnr}{100} \text{ or}$$

$$P\left(1 + \frac{nr}{100}\right)$$

or principal amount + interest

1) What is the simple interest for Rs. 10, 000 at the rate of 15% per annum for 2 years?

Ans: P = 10,000, n = 2 years, r = 15

$$\text{Interest} = \frac{Pnr}{100} = \frac{10{,}000 \times 2 \times 15}{100}$$

$$= \underline{\underline{\text{Rs. } 3,000}}$$

2) Find the total interest and amount of the end of 5th year for as 10,000 at 10% per annum, simple interest.

Ans: P = 10,000, n = 5 years, r = 10%

$$\text{Interest} = \frac{Pnr}{100} = \frac{10{,}000 \times 5 \times 10}{100}$$

$$= \underline{\underline{\text{Rs. } 5,000}}$$

Amount at the end

$$5^{th} \text{ year} = P\left(1 + \frac{nr}{100}\right)$$

$$= 10{,}000\left(1 + \frac{5 \times 10}{100}\right)$$

$$= 10{,}000\left(1 + \frac{50}{100}\right)$$

$$= 10{,}000\left(\frac{150}{100}\right)$$

$$= 10{,}000 \times 1.5 \qquad = \underline{\underline{15{,}000}}$$

3) Find the simple interest and amount for Rs. 25,000 at 10% p. a for 26 weeks.

Ans: P = 25,000 n = 26/52, r = 10%

$$\text{Interest} = \frac{Pnr}{100} = \frac{25,000 \times \frac{26}{52} \times 10}{100}$$

$$= \frac{25,000 \times ½ \times 10}{100}$$

$$= \frac{25,000 \times 5}{100} = \underline{\underline{1250}}$$

$$\text{Amount at the end} = P(1 + \frac{nr}{100})$$

$$= 25000\left(1 + \frac{\frac{26}{52} \times 10}{100}\right)$$

$$= 25000\left(1 + \frac{5}{100}\right)$$

$$= 25000\left(\frac{105}{100}\right)$$

$$= 25000 \times 1.05 = \underline{\underline{26250}}$$

4) Find the simple interest and amount for Rs. 50,000 at 7.5% p. a for 4 months.

Ans: P = 50,000, n = 4/12, r = 7.5%

$$\text{Simple Interest} = \frac{50,000 \times \frac{4}{12} \times 7.5}{100}$$

$$= \frac{50,000 \times 1/3 \times 7.5}{100}$$

$$= \frac{50,000 \times 2.5}{100} = \underline{\underline{1250}}$$

$$\text{Amount} = 5000\left(1 + \frac{\frac{4}{12} \times 7.5}{100}\right)$$

$$= 5000\left(1 + \frac{2.5}{100}\right)$$

$$= 5000\left(\frac{102.5}{100}\right)$$

$$= 5000 \times 1.025 = \underline{\underline{51250}}$$

5) Find the number of years in which a sum of money will double itself at 25% p. a, simple interest.

Ans: P = p, Amount = 2P, r = 25, n = ?

Amount $= P(1 + \frac{nr}{100})$

$2P = P(1 + \frac{nr}{100})$

i.e., $2 = (1 + \frac{nr}{100})$

$= 2 - 1 = \frac{nr}{100}$

$= 1 = \frac{nr}{100}$

nr = 100

r = 25, ∴ n = 4

number of years = 4

6) At what rate would a sum of money double in 20 years ?

Ans: P = p, A = 2p, n = 20, r = ?

Amount $= P(1 + \frac{nr}{100})$

$2P = P(1 + \frac{nr}{100})$

i.e., $2 = 1 + \frac{nr}{100}$

$= 2 - 1 = \frac{nr}{100}$

$= 1 = \frac{nr}{100}$

= nr = 100

n = 20, then r = 5

∴ Rate of interest = 5% per annum.

7) Find the number of years an amount of Rs. 8000 will take to become Rs. 12000 at 6% p. a. Simple interest.

Ans: P = 8000, A = 12000, r = 6, n = ?

Total interest 12000 – 8000 = 2000

$$\text{Interest} = \frac{Pnr}{100}$$

$$4000 = \frac{8000 \text{x n x } 6}{100}$$

4000 x 100 = 8000 x 6 x n

400000 = 48000n

48000n = 4,00,000

$$n = \frac{400000}{48000} = \underline{\underline{8.33 \text{ years}}}$$

8) Find the rate of interest at which an amount of Rs. 12000 will become Rs. 15000 at the end of 10th year.

Ans: A = 15000, P = 12000, n = 1, r = ?

Total interest 15000 – 12000 = 3000

$$\text{Interest} = \frac{Pnr}{100}$$

$$3000 = \frac{12000 \text{x } 10 \text{ x r}}{100}$$

3000 x 100 = 12000 x 10 x r

300000 = 120000r

$$r = \frac{300000}{120000} = 2.5$$

Rate of interest = $\underline{\underline{2.5\%}}$

9) A certain sum amounts to Rs. 678 in 2 years and to Rs. 736.50 in 3-5 years find the rate of interest and principal amount.

Ans: Amount for 2 years = 678

" 3-5 years = 736.50

$$\text{Amount} = P(1 + \frac{nr}{100})$$

$$678 = P(1 + \frac{2r}{100}) \quad \text{---------(1)}$$

$$736.50 = P(1 + \frac{3.5r}{100}) \quad \text{--------(2)}$$

Divide (1) by (2)

$= \frac{678}{736.50} = \frac{1+\frac{2r}{100}}{1+\frac{3.5r}{100}}$

$= \frac{678}{736.50} = \frac{100+2r}{100+3.5r}$

= 678(100 + 3.5r) = 736.50(100 + 2r)

= 67800 + 2373r = 73650 + 1473r

= 2373r – 1473r = 73650 – 67800

= 900r = 5850

= r = 5850/900 = 6.5

Substituting the value of r

$P(1+\frac{2r}{100}) = 678$

$P(1+\frac{2x\ 6.5}{100}) = 678$

$P(1+\frac{13}{100}) = 678$

$P(\frac{113}{100}) = 678$

P(1.13) = 678

P = 678/1.13 = 600

Rate of interest = 6.5%

Principal amount at the begining = 600

10) A person lends Rs. 1500, a part of it at 5% p.a. and the other part at 9% p.a. If he receives a total amount of interest of Rs. 162 at the end of 2 years. Find the amount lent at different rate of interest.

Ans: Let x is the Principal of 1st part

Then principal of 2nd part = 1500 – x

Total interest = 162

Interest = $\frac{Pnr}{100}$

Total interest = interest of 1st part and interest of 2nd part

$$162 = \frac{x \times 2 \times 5}{100} + \frac{(1500 - X) \times 2 \times 9}{100}$$

$$= \frac{10x}{100} + \frac{(1500 - x) \times 18}{100} = 162$$

$$= \frac{10x + (27000 - 18x)}{100} = 162$$

$10x + (27000 - 18x) = 162 \times 100$

$10x - 18x = 16200 - 27000$

$-8x = -10800$

$8x = 10800$

$x = 10800/8 = 1350$

Principal amount of 1st part = 1350

Principal amount of 2nd part = 150

Compound Interest

Compound interest means interest calculated on principal amount plus interest. Let 'p' be the principal 'r' be the rate of interest (compound) p.a., 'n' be the number of years then

$$\text{Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Total interest = A – P

1) Find CI on Rs. 25200 for 2 years at 10% p.a compounded annually?

Ans: P = 25200, r = 10, n = 2

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$= 25200\left(1 + \frac{10}{100}\right)^2$$

$$= 25200\left(\frac{110}{100}\right)^2$$

$= 25200 \times (1.10)^2$

$= 25200 \times 1.21 = 30492$

C1 = 30492 – 25200

= 5292

======

2) Find the Compound Interest Rs.10,000/- for 2½ years at 10% p.a..

Ans: P = 10,000 n = 2½ r = 10

Amount for 2 years = $p\left(1+\frac{r}{100}\right)^n$

$= 10{,}000\left(1+\frac{10}{100}\right)^2$

$= 10{,}000\left(\frac{110}{100}\right)^2$

$= 10{,}000 \times (1.1)^2$

= 10,000 x 1.21

= 12,100/-

Interest for 2 years = 2100

Interest for 6 months = $12100 \times \frac{10}{100} \times \frac{6}{12}$

= 605

Total interest for 2½ years = 2100 + 605

= 2,705/-
=======

3) X borrowed Rs.26,400/- from a bank to buy a scooter at the rate of 15% p.a. compounded yearly. What amount will be pay at the end of 2 years and 4 months to clear the loan.

Ans: p = 26,400/- r = 15

n = 2 years 4 months ($2\ ^1/_3\ years$)

Amount at the end of 2 years $= p\left(1+\frac{r}{100}\right)^n$

$= 26400\left(1+\frac{15}{100}\right)^2$

$= 26400\left(\frac{115}{100}\right)^2$

$= 26400\,(1.15)^2$

= 34,914

Interest for 4 months $= 34914 \times \frac{15}{100} \times \frac{4}{12}$

$= 1745.7$

Total amount at the end of 2 years and 4 months

ie $34914 + 1745.7 = 36659.7$

==========

4) Mr. A borrowed Rs.20,000/- from a person, but he could not repay any amount in a period of 4 years. So the lender demanded as 26500 which is the rate of interest charged.

Ans: Here interest charged on compound

P = 20,000 n = 4 A = 26500 r = ?

$$A = p\left(1 + \frac{r}{100}\right)^n$$

$$26500 = 20000\left(1 + \frac{r}{100}\right)^4$$

$$\frac{26500}{20000} = \left(1 + \frac{r}{100}\right)^4$$

$$1.325 = \left(1 + \frac{r}{100}\right)^4$$

$$\log 1.325 = 4 \log\left(1 + \frac{r}{100}\right)$$

$$0.1222 = 4 \log\left(1 + \frac{r}{100}\right)$$

$$\log\left(1 + \frac{r}{100}\right) = \frac{0.1222}{4}$$

$$\log\left(1 + \frac{r}{100}\right) = 0.03055$$

Antilog 0.03055 = 1.073

$$\left(1 + \frac{r}{100}\right) = 1.073$$

$$\frac{r}{100} = 1.073 - 1$$

$$\frac{r}{100} = 0.073$$

$$r = 100 \times 0.073 = 7.3\%$$

=======

5) The population of a country increases every year by 2.4% of the population at the beginning of first year. In what time will be population double itself? Answer to the nearest year?

Ans: p = p A = 2p r = 2.4 n = ?

$$A = p\left(1 + \frac{r}{100}\right)^n$$

$$2p = p\left(1 + \frac{2.4}{100}\right)^n$$

$$2p = p\left(\frac{102.4}{100}\right)^n$$

$$2p = p\,(1.024)^n$$

$$p = (1.024)^n$$

log 2 = n log 1.024

0.3010 = n x 0.0103

$$n = \frac{0.3010}{0.0103} = 29.22 = 30$$

===

6) The population of a city increases every year by 1.8% of the population at the beginning of that year, in how many years will the total increase of population be 30%?

Ans: p = p A = 1.3p r = 1.8 n = ?

$$A = p\left(1 + \frac{r}{100}\right)^n$$

$$1.3\,p = p\left(1 + \frac{1.8}{100}\right)^n$$

$$1.3p = p\left(\frac{101.8}{100}\right)^n$$

$$1.3\,p = p\,(1.018)^n$$

$$1.3 = (1.018)^n$$

log 1.3 = n log 1.018

0.1139 = n x 0.0076

$$n = \frac{0.1139}{0.0076} = 14.987$$

$$= 15$$

=====

7) In a certain population, the annual birth and death rates per thousand are 39.4 and 19.4 respectively. Find the number of years in which population will be doubled assuming that there is no emigration or immigration?

Ans: p = p A = 2p

$$r = \frac{39.4-19.4}{1000} \text{ x } 100 = 2\%$$

r = 2 n = ?

$$A = p\left(1+\frac{r}{100}\right)^n$$

$$2p = p\left(1+\frac{2}{100}\right)^n$$

$$2 = \left(1+\frac{2}{100}\right)^n$$

$$2 = p\,(1.02)^n$$

log 2 = n log 1.02

0.3010 = n x 0.0086

$$n = \frac{0.3010}{0.0086} = 35 \text{ years}$$

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COMPOUNDING HALF YEARLY OR QUARTERLY

- When interest is compounded half yearly, then $r = {}^{r}/_{2}$, n = 2n.
- When interest is compounded quarterly, then $r = {}^{r}/_{4}$, n = 4n.
- When interest is compounded monthly, then $r = {}^{r}/_{12}$, n = 12n.

1) Find the compound interest on Rs.50,000/- for 2 ½ years at 6% p.a. interest being compounded half yearly.

Ans: p = 50,000 n = 2 ½ x 2 = 5

$r = {}^{6}/_{2} = 3$

$$\text{Amount} = 50{,}000\left(1+\frac{3}{100}\right)^5$$

$$= 50{,}000\left(\frac{103}{100}\right)^5$$

$$= 50{,}000\,(1.03)^5 = 57964$$

C1 = 7964

========

2) Find the compound interest on Rs.60,000/- for 4 years, if interest is payable half yearly for due first 3 years at the rate of 8% p.a. and for the fourth year, the interest is being payable quarterly at the rate of 6% p.a.

Ans: Amount at in end of 3 years

$n = 3 \times 2 = 6, \qquad r = \frac{8}{2} = 4$

$p = 6{,}000$

$= 6{,}000\left(1 + \frac{4}{100}\right)^6$

$= 6{,}000\left(\frac{104}{100}\right)^6$

$= 6{,}000\,(1.04)^6$

$= 6{,}000 \times 1.2653$

$= 7592$

========

For last year

$n = 1 \times 4 = 4, \quad r = \frac{6}{2} = 1.5, \quad p = 7{,}592$

Amount at the end of 4^{th} year

$= 7592\left(1 + \frac{1.5}{100}\right)^4$

$= 7592\,(1.015)^4$

$= 7592 \times 1.0613 = 8057$

Interest $= 8057 - 6000 = 2057$

========

3) Find the effective rate of interest if interest is calculated at 10% p.a. half yearly?

Ans: Let $p = 100, \; n = 1 \times 2 = 2, \quad r = \frac{10}{2} = 5$

$A = p\left(1 + \frac{r}{100}\right)^n$

$= 100\left(1 + \frac{5}{100}\right)^2$

$= 100\left(\frac{105}{100}\right)^2$

$= 100 \times 1.1025 = 110.25$

C 1 $= 110.25 - 100 = 10.25$

Effective rate $= 10.25\%$ p.a.

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