MODULE - II

THEORY OF EQUATIONS

An equation is a statement of equality between two expressions. For eg:- $x + 2 = 5$. An equation contains one or more unknowns.

Types of Equations

1)Linear Equation

It is an equation when one variable is unknown. For example $2x + 3 = 7$

Practical Problems

1) Solve $2x + 3 = 7$

Ans : $2x = 7 - 3$

$$2x = 4, x = \frac{4}{2} = 2$$

2) Solve $3x + 4x = 35$

Ans : $7x = 35, x = \frac{35}{7} = 5$

3) Solve $4(x - 2) + 5(x - 3) - 25 = x + 8$

Ans : $= 4x - 8 + 5x - 15 - 25 = x + 8$

$= 4x + 5x - x = 8 + 8 + 15 + 25$

$8x = 56$

$x = \frac{56}{8} = 7$

==

4) $7x - 21 - 3x + 13 = 7 + 6x - 19$

Ans : $7x - 3x - 6x =$

$7 - 19 + 21 - 13$

$= {}^-2x = {}^-4$

$2x = 4$

$x = \frac{4}{2} = 2$

==

5) $^-23x + 14 - 7x + 16 = 10x - 17 + 3x + 4$

Ans : $^-23x - 7x - 10x - 3x = 17 + 4 - 14 - 16$

$-23x = -23$

$23x = 23$

$X = \dfrac{23}{23} = 1$

$==$

6) Find two numbers whose sum is 30 and difference is 4

Ans :   Let one number = x

then other number = $30 - x$

Numbers = $(30 - x) - x = 4$

$^-2x = 4 - 30$

$^-2x = -26$

$2x = 26$

$X = \dfrac{26}{3} = 13$

then numbers are 13, 17

$====$

7) Two third of a number decreased by 2 equals 4. Find the number

Ans : Let the number = x

Then $^2/_3(x) - 2 = 4$

$2x - 6 = 12$

$2x = 12 + 6$

$2x = 18$

$x = 9$

$==$

8) Solve $\dfrac{7x + 4}{x + 2} = \dfrac{-4}{3}$

Ans : $= 3(7x + 4) = {}^-4\,(x + 2)$

$= (21x + 12) = {}^-\,4x + {}^-8$

$21x + 4x = {}^-8 - 12$

$25x = {}^-20$

$X = \dfrac{-20}{25} = \dfrac{-4}{5}$

9) The ages of Hari and Hani are in the ratio of 4 : 5. Eight years from now, the ratio of their ages will be 5:6. Find their present age?

Ans : Let present age = $4x$ and $5\,x$

After 8 years $= \dfrac{4x + 8}{5x + 8} = \dfrac{5}{6}$

$= 6(4x + 8) = 5(5x + 8)$

$= 24x + 48 = 25x + 40$

$= 24x - 25x = 40 - 48$

$= -1x = -8$

$= x = 8$

Present ages of Hari and Hani are

Hari $= 4x = 4 \times 8 = 32$ years
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Hani $= 5x = 5 \times 8 = 40$ years
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2) Simultaneous equations in two unknowns

For solving the equations, firstly arrange the equations. For eliminating one unknown variable, multiply the equation 1 or 2 or both of them with certain amount and then deduct or add some equation with another, we get the value of one variable. Then substitute the value in the equation, we get the values of corresponding variable.

PRACTICAL PROBLEMS

1) Solve $3x + 4y = 7$

$\qquad 4x - 7 = 3$

Ans : $3x + 4y = 7$ ------ (1)
$\qquad 4x - y = 3$ ------- (2)

Multiply the equation 2 by 4, then

$$3x + 4y = 7 \text{ --------- (1)}$$
$$\underline{16x - 4y = 12}$$

Add $\qquad\qquad\qquad\qquad 19\,x = 19$

$$x = \dfrac{19}{19} = 1$$
$$==$$

Substitute to value of x

$\qquad 3x + 4y = 7$

$\qquad 3 \times 1 + 4y = 7$

$\qquad 3 + 4y = 7 = 4y = 7 - 3 = 4$

$\qquad y = \dfrac{y}{y} = 1$

2) $4x + 2y = 6$
$\quad 5x + y = 6$

Ans : $4x + 2y = 6$ ----------- (1)

        $5x + y = 6$ ---------- (2)

Multiply the equation 2 by 2, then

        $4x + 2y = 6$

        $\underline{10x + 2y = 12}$

        $^-6x = {}^-6$ (Deduct 1 – 2)

        $6x = 6$

        $x = \dfrac{6}{6} = 1$

        $5x + y = 6$

        $5 \times 1 + y = 6$

        $5 + y = 6, y = 6 - 5 = 1$
                          ==

Solve $y = 3(x + 1)$

        $4x = 4 + 1$

Ans : $y = 3x + 1$

        $4x = 4 + 1$

Arrange the equation

        $-3x + y = 3$ ----- (1)
        $\underline{4x - y = 1 ---------- (2)}$

        $1x = 4$ Add
        $x = 4$

Substituting the value of x

        $4x - y = 1$

        $16 - y = 1$

        $Y = 16 - 1 = 15$

        $X = 4, y = 15$
        =========

4) Solve $8x + 7y = 10$

$11x = 10(1-y)$

Ans : $8x + 7y = 10$ ------ (1)

        $11x = 10 - 10 y$

        $11x + 10 y = 10$ -------- (2)

Multiply equation (1) by 11 and (2) by 8

$$88x + 77y = 110$$
$$88x + 80y = 80$$
_____
$(1-2)$ $-3y = 30$

$$y = \frac{30}{-3} = ^-10$$

Substituting the value of y

   $8x + 7y = 10$

   $8x + 7 \times -10 = 10$

   $8x + -70 = 10$

   $8x = 10 + 70$

   $8x = 80, x = \frac{80}{8} = 10$

   $x = 10, y = -10$
   ==========

5) Solve $\dfrac{x-y}{2} = \dfrac{y-1}{3}$ and $\dfrac{3x-4y}{5} \times ^-10$

   $$\frac{x-y}{2} = \frac{y-1}{3}$$

   $= 3(x-y) = 2(y-1)$

   $= 3x - 3y = 2y - 2$

   $3x - 3y - 2y = -2$

   $3x - 5y = -2$ ---------- (1)

   $$\frac{3x-4y}{5} = x - 10$$

   $3x - 4y = 5(x-10)$

   $3x - 4y = 5x - 50$

   $3x - 5x - 4y = -50$

   $= 2x + 4y = 50$

   $= x + 2y = 25$ -------- (2)

Multiply equation (2) by 3

   $3x - 5y = ^-2$

   $3x + 6y = 75$

   $(1-2)\ -11y = -77$

   $$y = \frac{-77}{-11} = 7$$

Substituting the value

$x + 2y = 25$

$x + 2y = 25$

$x = 11$

====

$x = 11, y = 7$

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6) A man sells 7 horses and 8 cows at Rs. 2940/- and 5 horses and 6 cows at Rs. 2150/-. What is selling price of each ?

Ans : Let the selling price of horse = x

Cow = y

$7x + 8y = 2940$ ------- (1)

$5x + 6y = 2150$ --------(2)

Multiply equation (1) by 5 and 2 by 7

Then $3x + 40 y = 14700$

$35x + 42y = 15050$

(1-2) $-2y = -350$

$$y = \frac{-350}{-2} = 175$$

Substituting the value of y

$7x + 8y = 2940$

$7x + 8 \times 175 = 2940$

$7x = 2940 - 1400$

$7x = 1540$

$$x = \frac{1540}{7} = 220$$

Selling price of horse = *220*

===

Selling price of cow = 175

===

3) Simultaneous Equations in three unknowns

Firstly, eliminate one of the unknown from first two equations. Then eliminate the same unknown from second and third equations. Then we get two equations. Solve such equations, we get the values of x, y and z.

1)   Solve $4x + 2y - 32 = 2$

$$3x + 4y - 2z = 10$$

$$2x - 5y = 5$$

Ans: First consider first two equation and eliminate one unknown

$$4x + 2y - 3z = 2$$

$$3x + 4y - 2z = 10$$

For eliminating 2 multiply equation in 1 by 2 and 2 by 3, then

$$8x + 4y - 62 = 4$$

$$9x + 12y = 30$$

$$(2-1) \overline{x + 8y = 26} \qquad _____ (1)$$

Consider equation 2 and 3

$$3x + 4y - 2z = 10$$

$$2x - 5y + 4z = 5$$

On multiply xy equals 2 by 2

$$6x - 8y - 42 = 20$$

$$2x - 5y + 42 = 5$$

$$\overline{}$$

add $\quad 8x + 3y = 25 \qquad _____ (2)$

Solve the new equation 1 and 2

$$x + 8y = 26 \qquad _____(1)$$

$$8x + 3y = 25 \qquad _____ (2)$$

Multiply equation 1 by 8, then

$$8x + 64y = 208$$

$$8x + 3y = 25$$

$$\overline{}$$

$$(1-2) \qquad 61y = 183$$

$$Y = \frac{183}{61} = 3$$

Substitute value of Y

$$x + 8y = 26$$

$$x + 8 \times 3 = 26$$

$$x + 24 = 26$$

$$x = 26 - 24 = 2$$

Substitute the value of x, y,

$$4x + 2y - 3z = 2$$
$$4 \times 2 + 2 \times -3z = 2$$
$$8 + 6 - 3z = 2$$
$$14 - 3z = 2$$
$$3z = 14 - z$$
$$3z = 12$$
$$z = 12/3 = 4$$
$$x = 2, \ y = 3, \ z = 4$$
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4) Quadratic equations

The equation of the form ax^2 + bx+ c = 0 in which a, b, c are constant is called a quadratic equation in x. Here x is the unknown.

Solution of quadratic equations

There are three methods to solve a quadratic equation.

(1) Method by formula
(2) Method of factorization
(3) Method of completing the squre

Quadratic formula method

One general quadratic equation is ax^2 + bx + c = 0

Then $\qquad x = -b \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$

1)   Solve the equation $x^2 - x - 12 = 0$

Ans: a = 1, b= -1, c=-12

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\frac{\sqrt{-(-1)^2 \pm 4 \times 1 \times (-12)}}{2 \ \times \ 1}$$

$$1 \pm \frac{\sqrt{49}}{2}$$

$$1 \pm \frac{7}{2} = \frac{8}{12} , or \ \frac{-6}{2}$$

4 or -3
=======

2) Solve the equation 2x + $\dfrac{5}{x}$ = 7

Ans: Multiply the equation by x

Then

$2x^2 + 5 = 7x$

$2x^2 - 7x + 5 = 0$

a = 2, b= -7, c=5

$x = -b \pm \dfrac{\sqrt{b^2-4ac}}{2a}$

$-(-1) \pm \dfrac{\sqrt{1-4 \ 1 \times 1 \times -7}}{2 \times 1}$

$1 \pm \dfrac{\sqrt{1-- \ 28}}{2}$

$1 + \dfrac{\sqrt{29}}{2} \quad = \quad 1 \pm \dfrac{\sqrt{29}}{2}$

3) Solve the equation (x + 1) (x +2) − 3 = 0

Ans: $x^2 + 2x + x + 2 - 3 = 0$

$x^2 + 3x + 2 - 3 = 0$

$x^2 + 3x - 1 = 0$

a = 1, b = 3, c= −1

$x = -b \pm \dfrac{\sqrt{b^2-4ac}}{2a}$

$-3 \pm \dfrac{\sqrt{3^2-4 \times 1 \times 1}}{2 \times 1}$

$-3 \pm \dfrac{\sqrt{9 \ --4}}{2} \quad = \quad -3 \pm \dfrac{\sqrt{13}}{2}$

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4) Solve $x^4 - 10 x^2 + 9 = 0$

Ans: Let x^2 = y

Then equation =

$y^2 - 10y + 9 - 3 = 0$

$y = -b \pm \dfrac{\sqrt{b^2-4ac}}{1}$

a = 1, b = ⁻10, c= 29

$= (-10 \pm \dfrac{\sqrt{-10^2-4 \times 1 \times 9}}{2}$

$$10 \pm \frac{\sqrt{100-36}}{2}$$

$$10 \pm \frac{8}{2} = 9, 1$$

Y = 9, 1

$x^2 = y$, then $x = \sqrt{y}$

Y = 1, x= $\sqrt{1}$ = ± 1

Y = 9, x = $\sqrt{9}$ = ± 3

X = $^-$1, 1, 3, $^-$3

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5) $2x - 7\sqrt{x} + 5 = 0$

Answer = Let$\sqrt{x}$ = y, then equation

$$2y^2 - 74 + 5 = 0$$

$$y = {}^-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$${}^-7 \pm \frac{\sqrt{-7^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$7 \pm \frac{\sqrt{49 - 40}}{4}$$

$$7 \pm \frac{3}{4} = \frac{10}{4} \quad \text{or}$$

y = 4, or $\frac{-1}{2}$

y = 1, x = 1^2 = 1

$$y = \frac{10}{4} = X = \frac{10^2}{4^2} = \frac{100}{16} = \frac{25}{4}$$

x= 1, $\frac{25}{4}$

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6)   Solve $x^{10} - 33x^5 + 32 = 0$

Ans: Let $y = x^5$, Then equation

$$= y^2 - 33y + 32 = 0$$

Use quadratic formula

Y = 32, 1

$Y = 32$ then $x^2 = 32$

$= 2^5 = 32$

$\therefore x = 2$

$y = 1$ then $x^5 = 1$

$= 1^5 = 1, x = 1$

$X = 2, 1$

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7)   Solve $x + y = 10$

$xy = 24$

Ans: change to equation in the form of quadratic

$x + y = 10$

$x = 10 - y$

Substitute the value in second equation

$xy = 24$

$(10 - y) y = 24$

$= 10y - y^2 = 24$

$y^2 - 10 + 24 = 0$

Use quadratic formula

$y = -b \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$

$= (-10 \pm \dfrac{\sqrt{-10^2 - 4 \times 1 \times 24}}{2 \times 1}$

$10 \pm \dfrac{\sqrt{100 - 96}}{2}$

$10 \pm \dfrac{2}{2} = 6, 4$

when $y = 6,\ x = 4$

$y = 4,\ x = 6$

8) Simultaneous equations of two unknowns when one of them is quadratic and the other is linear

1) $x + y = 7$

$x^2 + y^2 = 25$

Answer

$x + y = 7$

$y = 7 - x$

Substistue the value of y y in the second equation, then

$$x^2 + (7 - x)^2 = 25$$

We know $(a - b)^2 = a^2 - 2ab + b^2$

$$x^2 + 7^2 - 2 \times 7 \times x + x^2 = 25$$

$$x^2 + 49 - 14x + x^2 = 25$$

$$x^2 + x^2 - 14x + 49 - 25$$

$$2x^2 - 14x + 24 = 0$$

Use quadratic formula

$$y = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a} =$$

$$=$$

$$= (-14 \pm \frac{\sqrt{-14^2 - 4 \times 2 \times 24}}{2 \times 2}$$

$$14 \pm \frac{\sqrt{4}}{4} = 14 \pm \frac{2}{4} = 4, 3$$

When $y = 4, x = 3$

$$Y = 3, x = 4$$

$$=========$$

2) Solve $x + y = 5$

$$2x^2 - y^2 - 10x - 2xy - 28 = 0$$

Ans: $y = 5 - x$

Substitute the value of y is equation (2)

$$2x^2 - (5 - x)^2 - 10x - 2x(5\text{-}x)$$

$$+ 28 = 0$$

$$= 3x^2\ 10x + 3 = 0$$

Use quadratic formula

$$X = 3 \text{ or } \frac{1}{3}$$

When $x = 3, y = 2$

When $x = \frac{1}{3}, y = \frac{14}{3}$