Measures of skewness

Skewness means lack of symmetry when a frequency distribution is not symmetrical, it is said t be asymmetrical or skewed. In the case of a skewed distribution, the mean, median and mode are not equal. Similarly for a skewed distribution $Q_1$ and $Q_3$ will not be equidistant from median. It is an asymmetrical distribution. It has a long tail on one side and a start tail on the other side.

A distribution is said to be skewed when:

(1) Mean, media and mode are not equal.

(2) $Q_1$ and $Q_3$ are not equidistant from median.

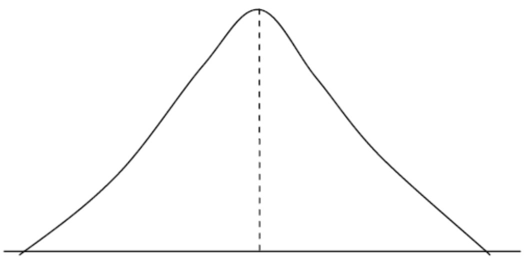
(3) Frequencies on either side of mode are not equal.

(4) The frequency curve has longer tail on the left side or on the right side.

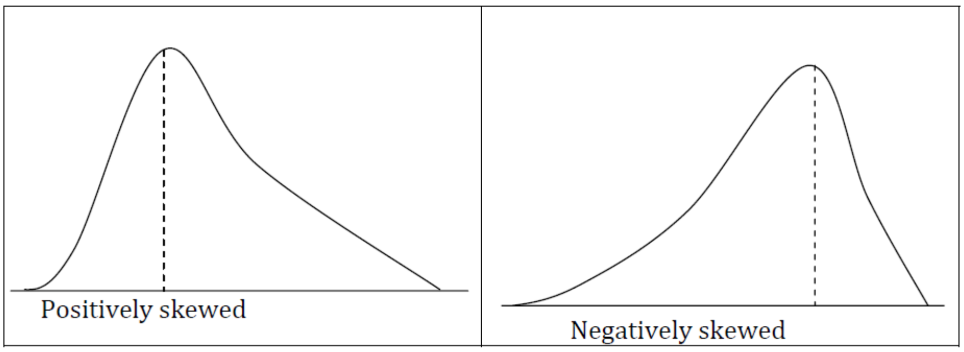
**Positive and Negative skewness**

Skewnwss may be either positive or negative. Skewness is said to be positive when the mean is greater than the median and median is greater than mode. More than half area falls to right side of the highest ordinate.

Swewness is said to be negative when the mean is less than median and the median is less than mode. In this case curve is skewed to the left more than half the area falls to the left of the highest ordinate.



Symmetric Distribution



Positively skewed

Negatively skewed

**Measures of skewness**

1) Karl Pearson's measure of skewness

$$\text{Skewness} = \frac{Mean - Median}{\sigma}$$

2) Bowley's measure of skewness

$$\text{Skewness} = \frac{Q_3 + Q_1 - 2Median}{Q_3 - Q_1}$$

3) Kelley's measure of skewness

$$\text{Skewness} = \frac{P_{90} + P_{10} - 2Median}{P_{90} - P_{10}}$$

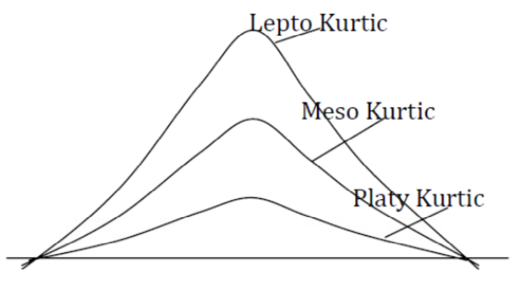
4) Measure of skewness Based on Moments

$$\text{Skewness} = \frac{M_3}{\sqrt{{M_2}^3}}$$

**Kurtosis**

Kurtosis is a measure of peakdness. It refers a distribution which is relatively fetker than the normal curve.

When a frequency curve is more peaked than the normal curve, it is called lepto kurtic and when it is more flat topped than the normal curve it is called platy kurtic. When a curve is neither peaked nor plat topped, it is called meso kurtic normal.




**Lorenz Curve**

Lorenz curve is a graphical method of studying dispersion. It is used in business to study the disparities of the distribution of wages, sales, production etc. In Economics it is useful to measure inequalities in the distribution of income.

It is a graph down to a frequency distribution. While drawing the graph, cumulative percentage values of frequencies on X axis and cumulative percentage values of the variable on Y axis.

**Index Numbers**

Index numbers is a statistical device for measuring the changes in group of related variables over a period of time.

**Uses or Importance of index numbers.**

1. Index numbers measure trend values.
2. Index numbers facilitate for policy decisions.
3. Index numbers help in comparing the standard of living.
4. It measures changes in price level.
5. Index numbers are economic barometers. The condition of the economy of a country to be known through construction of index numbers for different periods with regard to employment, literacy, agriculture industry, economics etc. Hence it can be termed as economic barometers.

**Limitations**

1. Index numbers are only approximate indicator.
2. All index numbers are not good for all purposes.
3. Index numbers are liable to be unissued.
4. Index numbers are specilised average and limitations of average also applicable to index numbers.

**Problems or Difficulties in the construction of index numbers**

1. Purpose of the index.
2. Selection of the lease period.
3. Selection of items.
4. Selection of an average
5. Selection of weights
6. Selection of appropriate source of data
7. Selection of suitable formula.

**Methods of constructing index numbers**

1. Unweighted index numbers.
2. Weighted index numbers.

**Unweighted or Simple index numbers**

Simple index numbers are those index numbers in which all items are treated as equally. Simple aggregate and simple average price relatives are the unweighted index numbers.

**(1) Simple Aggregate method**

$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$$

$P_{01}$ = index number

$P_1$ = Price for the current year

$P_1$ = Price for the base year.

**(2) Simple Average Price Relative Method**

$$\text{Price index} = \frac{\sum I}{n}$$

$I = \frac{P_1}{P_0}$ x 100, each items can be calculated.

**Weighted index numbers**

In this method quantity consumed is also taken into account.

Such index are

1. Weighted aggregate method
2. Weighted Average of price relatives

**Weighted aggregate method**

This method is based on the weight of the prices of the selected commodities.

Following are the commonly used methods:

1. Laspeyre's Method
2. Paasche's Method
3. Bowley-Dorbish Method
4. Fishers ideal method
5. Kelly's Methods

**Laspeyre's Method**

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

$p_1$ = Price of the current year

$q_0$ = Quantity of the base year

$p_0$ = Price of the base year

**Paasche's Method**

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

$q_1$ = Quantity of the current year

**Fishers Ideal Method**

$$P_{01} = \sqrt{L \times P} \times 100$$

L = Laspeyres method

P = Paasche's Method

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

**Bowley-Doribish Method**

$$P_{01} = \frac{L+P}{2}$$

**Kelly's Method**

$$P_{01} = \frac{\sum p_1 q}{\sum p_0 q} \times 100$$

$$q = \frac{q_0 + q_1}{2}$$

**Weighted Average Price Relative Method**

$$\text{Index number} = \frac{\sum IV}{\sum V}$$

V = Weight

$$I = \frac{P_1}{P_0} \times 100$$

1. Construct index numbers for 2012 on the basis of the price of 2010

| Commodities | Price in 2010 | Price in 2012 |
|---|---|---|
| A | 115 | 130 |
| B | 72 | 89 |
| C | 54 | 75 |
| D | 60 | 72 |
| E | 80 | 105 |

Answer

| Commodities | $P_0$ | $P_1$ |
|---|---|---|
| A | 115 | 130 |
| B | 72 | 89 |
| C | 54 | 75 |
| D | 60 | 72 |
| E | 80 | 105 |
| | **381** | **471** |
| | === | === |

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

$$= \frac{471}{381} \times 100 = 123.62$$

=====

2. Calculate simple index number by average relative method.

| Items | Price of the base year | Price of the current year |
|---|---|---|
| A | 5 | 7 |
| B | 10 | 12 |
| C | 15 | 25 |
| D | 20 | 18 |
| E | 8 | 9 |

Ans:

| Items | $P_0$ | $P_1$ | (ie $\frac{P_1}{P_0}$ x 100) |
|---|---|---|---|
| A | 5 | 7 | 140 |
| B | 10 | 12 | 120 |
| C | 15 | 25 | 166.7 |
| D | 20 | 18 | 90 |
| E | 8 | 9 | 112.5 |
| | | | **629.2** |
| | | | ==== |

$$\text{Index number} = \frac{\sum I}{n}$$

$$= \frac{629.6}{5} = 125.84$$

=====

3. Following are the data related with the prices and quantities consumed for 2010 and 2012.

| Commodity | 2010 | | 2012 | |
|---|---|---|---|---|
| | Price | Quantity | Price | Quantity |
| Rice | 5 | 15 | 7 | 12 |
| Wheat | 4 | 5 | 6 | 4 |
| Sugar | 7 | 4 | 9 | 3 |
| Tea | 52 | 2 | 55 | 2 |

Construct price index numbers by

(1) Laspeyre's method

(2) Paasche's method

(3) Bowly's – Dorbish method

(4) Fisher's method

**Answer**

| **Commodity** | $p_0$ | $q_0$ | $p_1$ | $q_1$ | $p_1q_0$ | $p_0q_0$ | $p_1q_1$ | $p_0q_1$ |
|---|---|---|---|---|---|---|---|---|
| Rice | 5 | 15 | 7 | 12 | 105 | 75 | 84 | 60 |
| Wheat | 4 | 5 | 6 | 4 | 30 | 20 | 24 | 16 |
| Sugar | 7 | 4 | 9 | 3 | 36 | 28 | 27 | 21 |
| Tea | 12 | 2 | 55 | 2 | 110 | 104 | 110 | 104 |
| | | | | | **281** | **227** | **245** | **201** |

(1) Laspeyre's Method

$p_{0\,1} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100 \quad = \frac{281}{227} \times 100$

$= \underline{\underline{123.79}}$

(2) Paascne's method

$p_{0\,1} = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100 \quad = \frac{245}{201} \times 100$

$= \underline{\underline{121.89}}$

(3) Bowley – Dorbish Method

$p_{0\,1} = \frac{L+P}{2} = \frac{123.79+121.89}{2}$

$= \underline{\underline{122.84}}$

(4) Fisher's Method

$p_{0\,1} = \sqrt{L \times P}$

$= \sqrt{123.79 \times 121.89} = 122.84$

4) Calculate index number of price for 2012 on the basis of 2010, from the data given below:

| Commodities | Weight | Price 2010 | Price 2012 |
|---|---|---|---|
| A | 40 | 16 | 20 |
| B | 25 | 40 | 60 |
| C | 5 | 2 | 2 |
| D | 20 | 5 | 6 |
| E | 10 | 2 | 1 |

**Answers**

Price Index Number = $\frac{\sum IV}{\sum V}$

| Commodities | V | $p_0$ | $p_1$ | i.e. $\frac{p_1}{p_0}$ x 100 | IV |
|---|---|---|---|---|---|
| A | 40 | 16 | 20 | 125 | 5000 |
| B | 25 | 40 | 60 | 150 | 3750 |
| C | 5 | 2 | 2 | 100 | 500 |
| D | 20 | 5 | 6 | 120 | 2400 |
| E | 10 | 2 | 1 | 50 | 500 |
| | **100** | | | | **12150** |

Index Number = $\frac{12150}{100}$ = $\underline{\underline{121.5}}$

5) Construct Price Index

| Commodities | Index | Weight |
|---|---|---|
| A | 350 | 5 |
| B | 200 | 2 |
| C | 240 | 3 |
| D | 150 | 1 |
| E | 250 | 2 |

**<u>Answers</u>**

| Commodities | V | I | IV |
|---|---|---|---|
| A | 5 | 350 | 1750 |
| B | 2 | 200 | 400 |
| C | 3 | 240 | 720 |
| D | 1 | 150 | 150 |
| E | 2 | 250 | 500 |
| | **13** | | **3520** |

Index Number = $\frac{\sum IV}{\sum V} = \frac{3520}{13} = \underline{\underline{270.77}}$

**Consumer Price index number of cost of Living index number or Retail Price index number**

Consumer Price index number is also known as copy of Living Index number or Retails Price index number. It is the ration of the monetary expenditures of an individual which secure him the standard of living or total utility in two situations differing only in respect of prices. It represents the average change in prices over a period of time, paid by the consumer for goods and services.

**Steps in the construction of Consumer Price Index**

1. Determination of the class people for whom the index number is to constructed.
2. Selection of Basic period
3. Conducting family budget enquiry
4. Obtaining price quotation
5. Selecting proper weights
6. Selection of suitable methods for constructing index.

**Methods of Constructing Consumer Price Index Number**

(1) Aggregate Expenditure Method

Cost of living Index number = = $\frac{\sum p_1 q_0}{\sum p_0 q_0}$ x 100

(2) Family Budget Method or Average Relative Method

Cost of Living Index = $\frac{\sum IV}{\sum V}$

1) Find cost of Living index

| | **Food** | **Rent** | **Clothes** | **Fuel** | **Miscellaniou** |
|---|---|---|---|---|---|
| Expenses on | 35% | 15% | 20% | 10% | 25% |
| Price 2010 | 150 | 30 | 75 | 25 | 40 |
| Price 2012 | 145 | 30 | 65 | 23 | 45 |

What changes the cost of living of 2012 as compare to 2010?

**Answer**

| Expenses | V | $p_0$ | $p_1$ | I | IV |
|---|---|---|---|---|---|
| Food | 35 | 150 | 145 | 96.67 | 3383.45 |
| Rent | 15 | 30 | 30 | 100 | 1500 |
| Cloth | 20 | 75 | 65 | 86.67 | 1733 |
| Fuel | 10 | 25 | 23 | 92 | 920 |
| Misc. | 20 | 40 | 45 | 112.50 | 2250 |
| | | | | | **9786.85** |

Cost of Living Index $= \frac{\sum IV}{\sum V} = \frac{9786.85}{100} = \cancel{97.87}$

**Time Series Analysis**

Time series is the arrangement of data according to the time of occurrence. It helps to find our the variations to the value of data due to changes in time.

**Importance**

1. It helps for understanding past behavior
2. It facilitates for forecasting and Planning
3. It facilitates comparison

**Components of Time Series**

1. Secular trend
2. Seasonal Variations
3. Cyclic Variations
4. Irregular Variations

**Secular Trend**

Trend may be defined as the changes over a long period of time. The significance of trend is greater when the period of time is very longer.

Following are the important method of measuring trend.

1. Graphic Method
2. Semi Average Method
3. Moving Average Method
4. Method of Least Squares

2) **Seasonal Variations**:- Seasonal Variations are measured for one calendar year. It is the variations which occur some degree of regularity. For example climate conditions, social customs etc.

3) **Cyclical Variations**:- Cyclical variations are those variation which occur on account of business cycle. They are Prosperity, Dectine, Depression and Recovery.

4) **Irregular fluctuations**:- One changes of variable could not be predicted due to irregular movements. Irregular movements are like changes in technology, war, famines, flood etc.

**Methods of Measuring Trend**

**(1) Graphic method:-** It is otherwise known as free hand method. This is the simplest method of measuring trend. Under this method original data are plotted on the graph paper. The plotted points should be joined, we get a curve. A straight line should be drawn through the middle area of the curve. Such line will describe tendency of the data.

**(2) Semi Average Method:-** The whole data are divided in to two parts and average of these are to be calculated. The two averages are to be plotted in the graph. The two points plotted should be joined so as to get a straight line. This line is called the ward live.

**(3) Method of Moving average:-** Under this method a series of successive average should be calculated from a series of values moving average may be calculated for 3,4,5,6 or 7 years periods.

The moving average can be calculated as follows:

For example 3 years moving average will be $\frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d+e}{3}$ and so on.

Five years moving average = $\frac{a+b+c+d+e}{5}, \frac{b+c+d+e+f}{5}$ and so on.

1) Compute 3 yearly moving average from the following data

| Years: | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Sales (in 000units) | 55 | 47 | 59 | 151 | 79 | 36 | 45 | 72 | 83 | 89 | 102 |

**Calculation of 3 yearly moving average**

| Year | Sales (in 000 units) | 3 yearly moving total | 3 yearly moving average |
|---|---|---|---|
| 2002 | 55 | ---------- | ----------- |
| 2003 | 47 | ---------- | ---------- |
| 2004 | 59 | 161 | 53.67 |
| 2005 | 151 | 257 | 85.67 |
| 2006 | 79 | 289 | 96.33 |
| 2007 | 36 | 216 | 58.67 |
| 2008 | 45 | 160 | 63.33 |
| 2009 | 72 | 153 | 51 |
| 2010 | 83 | 200 | 66.67 |
| 2011 | 89 | 244 | 81.33 |
| 2012 | 102 | 277 | 91.33 |

2) Calculate 5 yearly moving average

| Years: | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| income (in '000') | 161 | 127 | 152 | 143 | 144 | 167 | 182 | 179 | 152 | 163 | 159 |

**Answers**

| Year | Income (in 000) | Five yearly moving total | Five yearly moving average |
|---|---|---|---|
| 2000 | 161 | ---------- | ----------- |
| 2001 | 127 | ---------- | ---------- |
| 2002 | 152 | 727 | 145.4 |
| 2003 | 143 | 733 | 146.6 |
| 2004 | 144 | 788 | 157.6 |
| 2005 | 167 | 815 | 163 |
| 2006 | 182 | 824 | 164.8 |
| 2007 | 179 | 843 | 168.6 |
| 2008 | 152 | 835 | 167 |
| 2009 | 163 | ---------- | ----------- |
| 2010 | 159 | ---------- | ---------- |

**Calculation of moving average for every periods**

1) Calculate the six year moving average

| Years: | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Demand (in tones) | 105 | 120 | 115 | 110 | 100 | 130 | 135 | 160 | 155 | 140 | 145 |

**Answers**

| Year | Demand | 6 years moving total | 6 years moving average | Centered 6 years moving total | Centered 6 year moving average |
|---|---|---|---|---|---|
| 2000 | 105 | ------- | ------- | ------- | ------- |
| 2001 | 120 | -------- | -------- | -------- | -------- |
| 2002 | 115 | --------- | --------- | --------- | --------- |
| 2003 | 110 | 680 | 113.3 | 231.6 | 115.8 |
| 2004 | 100 | 710 | 118.3 | 243.3 | 121.65 |
| 2005 | 130 | 750 | 125 | 256.67 | 128.34 |
| 2006 | 135 | 790 | 131.67 | 268.34 | 134.17 |
| 2007 | 160 | 820 | 136.67 | 280.84 | 140.42 |
| 2008 | 155 | 865 | 144.17 | | |
| 2009 | 140 | | | | |
| 2010 | 145 | | | | |

4) **Method of Least Squares**

This is a popular method of obtaining trend line. The trend line obtained through this method is called line of best fit.

One trend line is represented as

$y = a + bx$

The value of **a** and **b** can be ascertained by solving the following two normal equations.

$\sum y = Na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$

Where **x** represents the time, **y** represents the value, **a** and **b** are constant and **N** represent total number.

When the middle year is taken as the origin, then $\sum x = 0$, then normal equation would be

$\sum xy = Na$

$\sum xy = b\sum x^2$

Hence a = $\frac{\sum xy}{\sum x^2}$

1) Following are the data related with the output of a factory for 7 years

| Years: | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|
| Output (in tones) | 47 | 64 | 77 | 88 | 97 | 109 | 113 |

Calculate the trend values through the method of least squares and also forecast the production 2013 and 2015.

**Answers**

| **Year t** | **Production y** | **x (t – 2009)** | **xy** | **$x^2$** |
|---|---|---|---|---|
| 2006 | 47 | -3 | -141 | 9 |
| 2007 | 64 | -2 | -128 | 4 |
| 2008 | 77 | -1 | -77 | 1 |
| 2009 | 88 | 0 | 0 | 0 |
| 2010 | 97 | 1 | 97 | 1 |
| 2011 | 109 | 2 | 218 | 4 |
| 2012 | 113 | 3 | 339 | 9 |
| | **595** | **0** | **308** | **28** |

Here $\sum x = 0$

Then $a = \frac{\sum y}{n} = \frac{595}{7} = 85$

$b = \frac{\sum xy}{\sum x^2} = \frac{308}{28} = 11$

$y = a + bx$

2006 - 85 + 11 x -3 = 52

2007 - 85 + 11 x -2 = 63

2008 - 85 + 11 x -1 = 74

2009 - 85 + 11 x 0 = 85

2010 - 85 + 11 x 1 = 96

2011 - 85 + 11 x 2 = 107

2012 - 85 + 11 x 3 = 118

**Production in 2013**

= 85 x 11 x 4 = 129 tonns

**Production in 2015**

= 85 x 11 x 6 = 151 tonns