

**MODULE - V****Weighted Mean**

Weighted means are obtained by taking in to account of weights. Each value is multiplied by its weight and total is divided by the total weight to get weighted mean.

$$\bar{x}_w = \frac{\sum wx}{\sum w}$$

$$\bar{x}_w = \text{weighted A.M.}$$

$$w = \text{weight}$$

$$x = \text{given variable}$$

**Median**

Median is the middle value of the series. When the series are arranged in the ascending order or descending order Median is a positional average.

**Calculation of Median****Individual series**

Firstly arrange the series.

$$\text{Median} = \text{Size of } \left(\frac{n+1}{2}\right)^{th} \text{ item.}$$

**Discrete series**

$$\text{Median} = \text{Size of } \left(\frac{n+1}{2}\right)^{th} \text{ item.}$$

**Continuous series**

$$\text{Median Class} = \frac{N}{2}$$

$$\text{Median} = L_1 + \frac{N/2 - c.f}{f} \times C$$

$$L_1 = \text{Lowerlimit of median class}$$

$$c.f = \text{cumulative frequency of preceding median class}$$

$$f = \text{frequency of median class}$$

$$C = \text{Class interval}$$

1) Find the median for the following data

4, 25, 45, 15, 26, 35, 55, 28, 48

**Answer:**

4, 15, 21, 25, 26, 28, 35, 45, 48, 55

$$\text{Median} = \left(\frac{N+1}{2}\right)^{\text{th}} \text{ item}$$

$$\left(\frac{9+1}{2}\right)^{\text{th}} \text{ item} = 5^{\text{th}} \text{ item}$$

$$\text{Median} = 28$$

2) Calculate median

25, 35, 15, 18, 17, 36, 28, 24, 22, 26

**Answer:**

15, 17, 18, 22, 24, 25, 26, 28, 35, 36

$$\text{Median} = \left(\frac{N+1}{2}\right)^{\text{th}} \text{ item}$$

$$\left(\frac{10+1}{2}\right)^{\text{th}} \text{ item}$$

$$= 5.5 \text{ item}$$

$$\text{Median} = \frac{5^{\text{th}} \text{ item} + 6^{\text{th}} \text{ item}}{2}$$

$$\frac{24 + 25}{2} = 24.5$$

3) Calculate median

Size :            5            8            10            15            20            25

Frequency :    3            12            8            7            5            4

**Answer:**

<u>Size</u>	<u>Frequency</u>	<u>Cf</u>
5	3	3
8	12	15
10	8	23
15	7	30
20	5	35
25	4	39

$$\text{Median} = \left(\frac{N+1}{2}\right)^{\text{th}} \text{ item}$$

$$\left(\frac{39+1}{2}\right)^{\text{th}} \text{ item} = 20^{\text{th}} \text{ item}$$

$$\text{Median} = 10$$

4) Find median from the following :

<u>Marks</u>	<u>No. of students</u>
0-5	29
10-15	195
15-20	241
20-25	117
25-30	52
30-35	10
35-40	6
40-45	2

**Answer :**

<u>Marks</u>	<u>f</u>	<u>c.f</u>
0-5	29	29
5-10	195	227
10-15	241	465
15-20	117	582
20-25	52	634
25-30	10	644
30-35	6	650
35-40	3	653
40-45	3	656
	-----	
	656	
	===	

$$\text{Median class} = N/2 = \frac{656}{2} = 328^{\text{th}} \text{ item}$$

$$\text{Median} = L_1 + \frac{N/2 - cf}{f} \times C$$

$$= 10 + \frac{328 - 224}{241} \times 5$$

$$= 12.2$$

$$===$$

**Mode**

Mode is the value of item of series which occurs most frequently.

**Mode in individual series**

In the case of individual series, the value which occurs more number of times is mode.

When no items appear more number of times than others, then mode is the ill defined. In this case :

$$\text{Mode} = 3 \text{ median} - 2 \text{ mean}$$

**Mode in discrete series**

In the case of discrete series, the value having highest frequency is taken as mode.

**Mode in continuous series**

Mode lies in the class having the highest frequency.

$$\text{Mode} = l_1 + \frac{(f_1 - f_0) \times C}{2f_1 - f_0 - f_2}$$

$l_1$  = lower limit of the modal class

$f_1$  = frequency of the modal class

$f_0, f_2$  = frequency of class preceding and succeeding modal class.

1) Find mode

1, 2, 5, 6, 7, 3, 4, 8, 2, 5, 4, 5

**Answer :**

Mode = 5

==

2) Find mode

4, 2, 6, 3, 8, 7, 9, 1

**Answer**

Mode is ill defined

Mode = 3 median - 2 mean

$$\bar{x} = \frac{\sum x}{n} = \frac{40}{8} = 5$$

Median : 1, 2, 3, 4, 6, 7, 9

$$\text{Median} = \frac{N+1^{th}}{2} \text{ item} = \frac{8+1}{2} = 4.5$$

$$\text{Median} = \frac{4^{th} + 5^{th} \text{ item}}{2} = \frac{4+6}{2} = 5$$

$$\text{Mode} = 3 \times 5 - 2 \times 5 = 5$$

==

3) Find mode

Size :	5	8	10	12	15	20	25
Frequency:	3	7	2	9	5	6	2

Mode = 12, since 12 has the highest frequency

4) Calculate mode

Size :	0-5	5-10	10-15	15-20	20-25	25-30
Frequency:	20	24	32	28	20	26

**Answer**

<u>Size</u>	<u>Frequency</u>
0-5	20
5-10	24
10-15	32 ---- -Models class
15-20	28
20-25	20
25-30	26

$$\begin{aligned}
 \text{Mode} &= l_1 + \frac{(f_1 - f_0) \times C}{2f_1 - f_0 - f_2} \\
 &= 10 + \frac{(32 - 24) \times 5}{2 \times 32 - 24 - 28} \\
 &= 10 + \frac{40}{12} \\
 &= 13.3 \\
 &===
 \end{aligned}$$

5) Calculate mean, median and mode

Marks	No. of students
Less than 10	4
Less than 20	9
Less than 30	15
Less than 40	18
Less than 50	26
Less than 60	30
Less than 70	38
Less than 80	50
Less than 90	54
Less than 100	55

**Answer :**

<u>Marks</u>	<u>Frequency</u>	<u>M</u>	<u>fm</u>	<u>c.f.</u>
0-10	4	5	20	4
10-20	5	15	75	9
20-30	6	25	150	15
30-40	3	35	105	18
40-50	8	45	360	26
50-60	4	55	220	30
60-70	8	65	520	38
70-80	12	75	900	50
80-90	4	85	340	54
90-100	1	95	95	55

Mean

$$\begin{aligned} \bar{x} &= \frac{\sum fm}{N} \\ &= \frac{2785}{55} \\ &= 50.63 \\ &==== \end{aligned}$$

Median

$$\begin{aligned} &= \frac{N^{th}}{2} \text{ item} \\ &= \frac{55^{th}}{2} \text{ item} \\ &= 27.5^{th} \text{ item} \\ &= l_1 + \frac{N/2 - c.f}{f} \times C \\ &= 50 + \frac{27.5 - 26}{4} \times 10 \\ &= 50 + \frac{1.5}{4} \times 10 \\ &= 73.33 \\ &==== \end{aligned}$$

6) Calculate mean, median and mode

Marks	No. of students
More than 0	80
More than 10	77
More than 20	72
More than 30	65
More than 40	55
More than 50	43
More than 60	28
More than 70	16
More than 80	10
More than 90	8

**Answer**

X	f	m	fm	c.f
0-10	3	5	15	3
10-20	5	15	75	8
20-30	7	25	175	15
30-40	10	35	350	25
40-50	12	45	540	37
50-60	15	55	825	52
60-70	12	65	780	64
70-80	6	75	450	70
80-90	2	85	170	72
90-100	8	95	760	80

Mean

$$\begin{aligned}\bar{X} &= \frac{\sum fm}{N} \\ &= \frac{4140}{8} \\ &= 51.5 \\ &===\end{aligned}$$

$$\begin{aligned}\text{Median} &= 80/2^{\text{th}} \text{ item} \\ &= 40^{\text{th}} \text{ item} \\ &= l_1 + \frac{N/2 - c.f}{f} \times C \\ &= 50 + \frac{40 - 37}{15} \times 10 \\ &= 50 + \frac{3}{15} \times 10 \\ &= 52 \\ &==\end{aligned}$$

$$\begin{aligned}
 \text{Mode} &= l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times C \\
 &= 50 + \frac{15 - 12}{2 \times 15 - 12 - 12} \times 10 \\
 &= 50 + \frac{3}{30 - 12 - 12} \times 10 \\
 &= 50 + \frac{3}{6} \times 10 \\
 &= 55 \\
 &==
 \end{aligned}$$

**Geometric Mean**

Geometric mean is defined as the  $n^{\text{th}}$  root of the product of those in values.

$$\text{G.m} = \text{Antilog} \left( \frac{\sum \log x}{n} \right)$$

**G.M in Individual series**

$$\text{G.M} = \text{Antilog} \left( \frac{\sum \log x}{n} \right)$$

**G.M in Discrete series**

$$\text{G.M} = \text{Antilog} \left( \frac{\sum f \log x}{n} \right)$$

G.M in continuous series

$$\text{G.m} = \text{Antilog} \left( \frac{\sum f \log x}{n} \right)$$

x = midpoint of x

1) Find Geometric mean of the following

57.5, 87.75, 53.5, 73.5, 81.75

**Answer:**

<b>X</b>	<b>logx</b>
57.5	1.7597
87.75	1.9432
53.5	1.7284
73.5	1.8663
81.75	1.9125
	9.2101
	====



$$\begin{aligned}
 \text{G.M.} &= \text{Antilog} \left( \frac{\sum \log x}{n} \right) \\
 &= \text{Antilog} \left( \frac{9.2101}{5} \right) \\
 &= \text{Antilog} (1.84202) \\
 &= 69.51 \\
 &====
 \end{aligned}$$

2) Find the G.M 2, 4, 8, 12, 16, 24

X	logx
2	0.3010
4	0.6021
8	0.9031
12	1.0792
16	1.2041
24	1.3802
	5.4697

$$\begin{aligned}
 \text{G.M.} &= \text{Antilog} \left( \frac{\sum \log x}{n} \right) \\
 &= \text{Antilog} \left( \frac{5.4697}{6} \right) \\
 &= \text{Antilog} (.9116) \\
 &= 8.158 \\
 &====
 \end{aligned}$$

3) Find G.M from the following data

Size :	5	8	10	12
Frequency:	2	3	4	1

Ans:

X	f	logX	f logx
5	2	.6990	1.3980
8	3	.9031	2.7093
10	4	1.0000	4.0000
12	1	1.0792	1.0792
	10		9.1865

$$\begin{aligned}
 \text{G.M.} &= \text{Antilog} \left( \frac{\sum \log x}{N} \right) \\
 &= \text{Antilog} \left( \frac{9.1865}{10} \right) \\
 &= \text{Antilog} (.91865) \\
 &= 8.292 \\
 &====
 \end{aligned}$$

4) Calculate G.M.

Daily Income ( ₹ )	0-20	20-	40-	60-80	80-
		40	60		100
No. of workers	5	7	12	8	4

**Answer :**

X	f	xf	logx	f logx
0-20	5	10	1.0000	5.0000
20-40	7	20	1.4771	10.3397
40-60	12	30	1.6990	20.3880
60-80	8	40	1.8451	14.7608
80-100	4	50	1.9542	7.8168
	36			58.3053

$$\begin{aligned}
 \text{G.M.} &= \text{Antilog} \left( \frac{\sum \log x}{N} \right) \\
 &= \text{Antilog} \left( \frac{58.3053}{36} \right) \\
 &= \text{Antilog } 1.6195916 \\
 &= 41.65 \\
 &====
 \end{aligned}$$

**Harmonic Mean**

Harmonic mean is defined as the reciprocal of the mean of the reciprocals of those values. It applied in averaging rates, times etc.

$$\text{H.M} = \frac{n}{\sum \frac{1}{x}}$$

H.M in Discrete series

$$\text{H.M} = \frac{N}{\sum f \left( \frac{1}{x} \right)}$$

H.M in continuous series

$$\text{H.M} = \frac{N}{\sum f \left( \frac{1}{x} \right)}$$

x = midpoint of x

1) Calculate H.M. from the following

1) Find the H.M.

2, 3, 4, 5

**Answer:**

x	$\frac{1}{x}$
2	0.5
3	0.33
4	0.25
5	0.20
	1.28

$$\begin{aligned}
 \text{H.M.} &= \left( \frac{n}{\sum \frac{1}{x}} \right) \\
 &= \frac{4}{1.28} \\
 &= 3.125 \\
 &====
 \end{aligned}$$

2) Find the H.M.

Size	6	10	14	18
F	20	40	30	10

Answer :

Size	f	$\frac{1}{x}$	$f(1/x)$
6	20	0.1667	3.334
10	40	0.1000	4.000
14	30	0.0714	2.142
18	10	0.0556	0.556
	100		10.032

$$\text{H.M} = \frac{N}{\sum f(1/x)} = \frac{100}{10.032} = 9.97$$

===

3) From the following data, calculate the value of HM?

Income ( ₹ )	No. of persons
10 - 20	4
20 - 30	6
30 - 40	10
40 - 50	7
50 - 60	3

Ans:

Income ( ₹ )	f	x in m	$\frac{1}{x}$	$f\left(\frac{1}{x}\right)$
10 - 20	4	15	0.667	0.2666
20 - 30	6	25	0.0400	0.2400
30 - 40	10	35	0.0286	0.2857
40 - 50	7	45	0.0222	0.1556
50 - 60	3	55	0.0182	0.0545
	30			1.0023

$$HM = \frac{N}{\sum f\left(\frac{1}{x}\right)} = \frac{30}{1.0023} = 29.93$$

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**MEASURES OF DISPERSION OR VARIABILITY**

Dispersion means a measure of the degree of deviation of data from the central value.

Measures of Dispersion are classified into (1) Absolute Measures  
(2) Relative Measures.

Absolute Measures of dispersion are expressed in the same units in which data are collected. They measure variability of series. Various absolute measures are:

- (i) Range
- (ii) Quartile Deviation
- (iii) Mean Deviation
- (iv) Standard Deviation

Relative measure is also called coefficient of dispersion. They are useful for comparing two series for their variability. Various relative measures are:

- (i) Coefficient Range
- (ii) Coefficient of Quartile Deviation
- (iii) Coefficient of Mean Deviation
- (iv) Coefficient of Variation

**RANGE**

The range of any series is the difference between the highest and the lowest values in the series.

$$\text{Range} = H - L$$

H = Highest variable

L = Lowest variable

$$\text{Coefficient of Range} = \frac{H-L}{H+L}$$

- 1) Find the Range and Coefficient of Range.

75, 29, 96, 15, 7, 8, 11, 7, 49

Ans:

$$\text{Range} = H - L$$

$$= 96 - 74 = 92$$

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$$\text{Coefficient of Range} = \frac{H-L}{H+L} = \frac{96-74}{96+74} = \frac{92}{170} = 0.92$$

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- 2) Find Range and Coefficient of Range.

Wages	5	10	15	20	25	30
No. of employees	2	5	6	7	4	6

Ans:

$$\text{Range} = H - L$$

$$= 30 - 5 = 25$$

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$$\text{Coefficient of Range} = \frac{H-L}{H+L} = \frac{30-5}{30+5} = \frac{25}{35} = 0.71$$

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3) Find out Range and Coefficient of Range.

Marks	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
No. of Students	8	12	20	7	3

Ans:

Marks	f
19.5 – 29.5	8
29.5 – 39.5	12
39.5 – 49.5	20
49.5 – 59.5	7
59.5 – 69.5	3

$$\text{Range} = H - L$$

$$= 69.5 - 19.5 = 50$$

$$\text{Coefficient of Range} = \frac{H-L}{H+L} = \frac{69.5-19.5}{69.5+19.5} = \frac{50}{89} = 0.56$$

### QUARTILE DEVIATION

Quartile Deviation is defined as the half distance between the third and first quartiles.

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of Quartile Deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

### Quartile Deviation in Individual Series

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = \text{size of } \frac{n+1}{4} \text{ th Item}$$

$$Q_3 = \text{size of } 3 \left( \frac{n+1}{4} \right) \text{ th item}$$

**Quartile Deviation in Discrete Series**

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = \text{size of } \frac{N+1}{4} \text{ th Item}$$

$$Q_3 = \text{size of } 3 \left( \frac{N+1}{4} \right) \text{ th item}$$

**Quartile Deviation in Continuous Series**

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = \text{size of } \frac{N}{4} \text{ th Item}$$

$$Q_3 = \text{size of } 3 \left( \frac{N}{4} \right) \text{ th item}$$

$$\text{Then, } Q_1 = L_1 + \frac{\frac{N}{4} - c.f}{f} \times c$$

$$Q_3 = L_1 + \frac{3 \left( \frac{N}{4} \right) - c.f}{f} \times c$$

4) Calculate Quartile Deviation from the following:

25, 15, 30, 45, 40, 20, 50

Also find coefficient of quartile deviation.

*Ans:* Arrange the series, then

15, 20, 25, 30, 40, 45, 50

$$\begin{aligned} Q_1 &= \frac{n+1}{4} \text{ th Item} = \frac{8}{4} = 2^{\text{nd}} \text{ Item} \\ &= 20 \end{aligned}$$

$$\begin{aligned} Q_3 &= 3 \left( \frac{n+1}{4} \right) \text{ th item} = 3 \times 2 = 6^{\text{th}} \text{ Item} \\ &= 45 \end{aligned}$$

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{45 - 20}{2} = 12.5$$

$$\text{Coefficient of Quartile Deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{25}{45 + 20} = \frac{20}{65} = 0.385$$

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## Basic Numerical Skills

2) Find Quartile Deviation and Coefficient of Quartile Deviation.

23, 25, 8, 10, 9, 29, 45, 85, 10, 16

Ans: Arrange the series, then

8, 9, 10, 10, 16, 23, 25, 29, 45, 85

$$Q_1 = \text{size of } \frac{n+1}{4} \text{th Item} = \frac{10+1}{4} \text{th Item} = 2.75 \text{th Item}$$

$$\text{i.e., } 2^{\text{nd}} \text{ Item} + .75 (3^{\text{rd}} \text{ Item} - 2^{\text{nd}} \text{ Item})$$

$$= 9 + .75 (10 - 9)$$

$$= 9 + .75 \times 1 = 9.75$$

$$Q_3 = \text{size of } 3 \left( \frac{n+1}{4} \right) \text{th item}$$

$$= 3 \times 2.75 = 8.25 \text{th Item}$$

$$\text{i.e. } 8^{\text{th}} \text{ item} + .25(9^{\text{th}} \text{ Item} - 8^{\text{th}} \text{ Item})$$

$$= 29 + .25 (45 - 29)$$

$$= 29 + .25 \times 16$$

$$= 29 + 4 = 33$$

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{33 - 9.75}{2} = 11.625$$

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$$\text{Coefficient of Quartile Deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{33 - 9.75}{33 + 9.75} = 0.54$$

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3) Find the value of Quartile Deviation and coefficient of Quartile Deviation?

Marks	25	30	40	50	60	70	80	90
No. of Students	4	7	12	8	9	15	7	3



Ans:

x	f	c.f.	
25	4	4	
30	7	11	
40	12	23	Q <sub>1</sub>
50	8	31	
60	9	40	
70	15	55	Q <sub>3</sub>
80	7	62	
90	3	65	
	65		

$$Q_1 = \frac{n+1}{4} \text{th Item} = \frac{65+1}{4} \text{th Item} = 16.5 \text{th Item}$$

$$Q_3 = 3 \left( \frac{n+1}{4} \right) \text{th item} = 3 \times 16.5 = 49.5 \text{th Item}$$

$$Q_1 = 45$$

$$Q_3 = 70$$

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{70 - 40}{2} = 15 \text{ marks}$$

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$$\text{Coefficient of Quartile Deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{70 - 40}{70 + 40} = 0.27$$

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4) Compute Quartile Deviation and coefficient of Quartile Deviation?

x	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
f	5	12	15	9	10	3

Ans:

x	f	c.f.
0 - 10	5	5
10 - 20	12	17
20 - 30	15	32
30 - 40	9	41
40 - 50	10	51
50 - 60	3	54
	54	

$$Q_1 = \text{size of } \frac{N}{4}^{\text{th}} \text{ Item} = \frac{54}{4}^{\text{th}} \text{ Item} = 13.5^{\text{th}} \text{ Item}$$

Which lies in 10 - 20 , then

$$\begin{aligned} Q_1 &= L_1 + \frac{\frac{N}{4} - c.f}{f} \times c \\ &= 10 + \frac{13.5 - 5}{12} \times 10 \\ &= 10 + \frac{8.5}{12} \times 10 \\ &= 10 + \frac{85}{12} = 17.08 \\ &\quad \text{=====} \end{aligned}$$

$$\begin{aligned} Q_3 &= 3 \left( \frac{N}{4} \right)^{\text{th}} \text{ item} \\ &= 3 \times 13.5 = 40.5^{\text{th}} \text{ Item} \end{aligned}$$

Which lies in 30 - 40, then

$$\begin{aligned} Q_3 &= L_1 + \frac{3 \left( \frac{N}{4} \right) - c.f}{f} \times c \\ &= 30 + \frac{40.5 - 32}{9} \times 10 \\ &= 30 + \frac{8.5}{9} \times 10 \\ &= 30 + \frac{85}{9} = 39.44 \\ &\quad \text{=====} \end{aligned}$$

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{39.44 - 17.08}{2} = \frac{22.36}{2} = 11.18 \text{ marks}$$

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$$\begin{aligned} \text{Coefficient of Quartile Deviation} &= \frac{Q_3 - Q_1}{Q_3 + Q_1} \\ &= \frac{39.44 - 17.08}{39.44 + 17.08} \\ &= \frac{22.36}{56.52} = 0.396 \end{aligned}$$

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**MEAN DEVIATION**

Mean Deviation is defined as the arithmetic mean of deviations of all the values in a series from their average. The average may be mean, median or mode.

$$\text{Mean Deviation} = \frac{\sum |d|}{n}$$

Where  $|d|$  = deviation from an average without sign

**Mean Deviation in Individual Series**

$$\text{Mean Deviation} = \frac{\sum |d|}{n}$$

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean Deviation}}{\text{Average}}$$

Average = Mean, Median or Mode from which the deviation is taken

**Mean Deviation in Discrete Series**

$$\text{Mean Deviation} = \frac{\sum f|d|}{N}$$

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean Deviation}}{\text{Average}}$$

**Mean Deviation in Continuous Series**

$$\text{Mean Deviation} = \frac{\sum f|d|}{N}$$

1) Calculate Mean Deviation from the following.

14, 15, 23, 20, 10, 30, 19, 18, 16, 25, 12

Ans:

Arrange the data

10, 12, 14, 15, 16, 18, 19, 20, 23, 25, 30

Median = size of  $\frac{11+1}{2}$  item

= 6<sup>th</sup> Item = 18

X	d  ie. X - median
14	4
15	3
23	5
20	2
10	8
30	12
19	1
18	0
16	2
25	7
12	6
	50

$$\text{Mean Deviation} = \frac{\sum |d|}{n} = \frac{50}{11} = 4.54 \text{ marks}$$

=====

2) Calculate Mean Deviation from the following data:

Size of item	6	7	8	9	10	11	12
Freequency	3	6	9	13	8	5	4

Ans:

Size	f	c.f	d	f  d
6	3	3	3	9
7	6	9	2	12
8	9	18	1	9
9	13	31	0	0
10	8	39	1	8
11	5	44	2	10
12	4	48	3	12
	48			60

$$\text{Median} = \frac{48 + 1}{2} \text{th item} = 24.5$$

$$\text{Median} = 9$$

$$= 18$$

$$\text{Mean Deviation} = \frac{\sum f|d|}{N} = \frac{60}{48} = 1.25$$

=====

3) Calculate the Mean Deviation from the following data:

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Freequency	18	16	15	12	10	5	2	2

Ans:

x	f	m	c.f.	d  ie. X - median	f d
0 - 10	18	5	18	19	342
10 - 20	16	15	34	9	144
20 - 30	15	25	49	1	15
30 - 40	12	35	61	11	132
40 - 50	10	45	71	21	210
50 - 60	5	55	76	31	155
60 - 70	2	65	78	41	82
70 - 80	2	75	80	51	102
	80				1182

$$\begin{aligned} \text{Median} &= \frac{N}{2} \text{th Item} \\ &= \frac{80}{2} \text{th Item} = 40^{\text{th}} \text{ Item} \end{aligned}$$

Which lies on 20 - 30

$$\begin{aligned} \text{Median} &= 20 + \frac{40-34}{15} \times 10 \\ &= 20 + \frac{6}{15} \times 10 \\ &= 24 \end{aligned}$$

$$\text{Mean Deviation} = \frac{\sum f|d|}{N} = \frac{1182}{80} = 14.775$$

=====

**STANDARD DEVIATION**

Standard Deviation is defined as the square root of the mean of the squares of the deviations of individual items from their arithmetic mean. It is denoted by  $\sigma$  (sigma).

$$\sigma = \frac{\sqrt{\sum(x - \bar{x})^2}}{n}$$

**Standard Deviation in Individual Series**

$$\sigma = \frac{\sqrt{\sum(x-\bar{x})^2}}{n} \text{ or } \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\text{Coefficient of variation} = \frac{\sigma}{\bar{x}} \times 100$$

**Standard Deviation in Discrete Series**

$$\sigma = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$$

Shortcut method:

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$d = x - A$$

**Standard Deviation in Continuous Series**

(i) Direct Method:

$$\sigma = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$$

$x$  = mid point of  $X$

(ii) Shortcut method:

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$d = m - A$  or  $x - A$

(iii) Step Deviation method:

$$\sigma = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times C$$

$d' = \frac{d}{c}$ ,  $c$  = class interval.

**VARIANCE**

Variance is defined as the mean of the squares of the deviations of all the values in the series from their mean. It is the square root of the Standard Deviation.

$$\text{Variance} = \sigma^2$$

1) Compute S.D

4, 8, 10, 12, 15, 9, 7, 7

Ans:

X	X <sup>2</sup>
4	16
8	64
10	100
12	144
15	225
8	81
7	49
7	49
<hr style="width: 50%; margin: 0 auto;"/> 72	<hr style="width: 50%; margin: 0 auto;"/> 728

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\begin{aligned}\sigma &= \sqrt{\frac{728}{8} - \left(\frac{72}{8}\right)^2} \\ &= \sqrt{91 - 9^2}\end{aligned}$$

$$\sigma = \sqrt{91 - 81} = \sqrt{10}$$

$$= 3.16$$

===

2) Find the S.D and C.V

10, 12, 80, 70, 60, 100, 0, 4

Ans:

<b>X</b>	<b>X<sup>2</sup></b>
10	100
12	144
80	6400
70	4900
60	3600
100	10000
0	0
4	16
<u>336</u>	<u>25160</u>

$$\begin{aligned}
 \sigma &= \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2} \\
 &= \sqrt{\frac{25160}{8} - \left(\frac{336}{8}\right)^2} \\
 &= \sqrt{3145 - (42)^2} \\
 &= \sqrt{3145 - 1764} = \sqrt{1381} \\
 &= 37.16 \\
 &====
 \end{aligned}$$

C.V.	$= \frac{\sigma}{\bar{X}} \times 100$
$\bar{X}$	$= \frac{336}{8} = 42$
C.V	$= \frac{37.16}{42} \times 100 = 88.48$ ====

3) Find out S.D

Production in tones :	50	100	125	150	200	250	300
No. of factories:	2	5	7	12	9	5	3



Ans:

X	f	d(x-A)	d <sup>1</sup>	d <sup>12</sup>	fd <sup>1</sup>	fd <sup>12</sup>
50	2	-100	-4	16	-8	32
100	5	-50	-2	4	-10	20
125	7	-25	-1	1	-7	7
150	12	0	0	0	0	0
200	9	50	2	4	18	36
250	5	100	4	16	20	80
300	3	150	6	36	18	108
	<b>43</b>				<b>31</b>	<b>283</b>

$$A = 150$$

$$d^1 = \frac{d}{25}$$

$$\begin{aligned} \sigma &= \sqrt{\frac{\sum fd^{12}}{N} - \left(\frac{\sum fd^1}{N}\right)^2} \times C \\ &= \sqrt{\frac{283}{43} - \left(\frac{31}{43}\right)^2} \times 25 \\ &= \sqrt{6.58 - 0.52} \times 25 \\ &= \sqrt{6.06} \times 25 = 2.46 \times 25 \\ &= 61.5 \\ &==== \end{aligned}$$

4) Compute the S.D from the following

Expenditure (Rs):	100-200	200-300	300-400	400-500	500-600
No. of families	30	20	40	5	10

Ans:

X	f	m	d	d <sup>1</sup>	d <sup>12</sup>	fd <sup>1</sup>	fd <sup>12</sup>
100-200	30	150	-200	-2	4	-60	120
200-300	20	250	-100	-1	1	-20	20
300-400	40	350	0	0	0	0	0
400-500	5	450	100	1	1	5	5
500-600	10	550	200	2	4	20	40
	<b>105</b>					<b>-55</b>	<b>185</b>

$$d = m - A$$

$$d^1 = d/100$$

$$\begin{aligned} \sigma &= \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times C \\ &= \sqrt{\frac{185}{105} - \left(\frac{-55}{105}\right)^2} \times 100 \\ &= 122 \\ &==== \end{aligned}$$

5) The scores of the batsmen A and B the six innings during a certain match are as follows.

Batsman A:            10    12    80    70    60    100    0    4

Batsman B:            8    9    7    10    5    9    10    8

- (i) Find which of the two batsman is more consistant in scoring.
- (ii) Find who is more efficient batchman.

Ans:

Batsman A		Batsman B	
X	X <sup>2</sup>	X	X <sup>2</sup>
10	100	8	64
12	144	9	81
80	6400	7	49
70	4900	10	100
60	3600	5	25
100	10000	9	81
0	0	10	100
4	16	8	64
<b><u>336</u></b>	<b><u>25160</u></b>	<b><u>66</u></b>	<b><u>564</u></b>

(i) For finding consistent, C.V is calculated

$$C.V = \frac{\sigma}{\bar{X}} \times 100$$

Batsman A

$$\bar{X} = \frac{336}{8} = 42$$

Batsman B

$$\bar{X} = \frac{66}{8} = 8.25$$

$$\sigma = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

$$\begin{aligned} \sigma &= \sqrt{\frac{25160}{8} - \left(\frac{236}{8}\right)^2} \\ &= 37.16 \\ &==== \end{aligned}$$

$$\begin{aligned} \sigma &= \sqrt{\frac{564}{8} - \left(\frac{66}{8}\right)^2} \\ &= 1.562 \\ &==== \end{aligned}$$

$$\begin{aligned} C.V &= \frac{37.16}{42} \times 100 \\ &= 88.48 \\ &==== \end{aligned}$$

$$\begin{aligned} \sigma &= \sqrt{\frac{564}{8} - \left(\frac{66}{8}\right)^2} \\ &= 18.93 \\ &==== \end{aligned}$$

B is more consistent since C.V. is less.

(ii) For finding more efficient, average is taken

$$A = 42$$

$$B = 8.25$$

Batsman A is more consistent since he has greater average.

Merits of S.D

1. S.D. is based on all the values of a series.
2. It is rigidly defined
3. It is capable of further mathematical treatment.
4. It is not much affected by sampling fluctuations.

Demerits

1. It is difficult to calculate.
2. Signs of the deviations are not ignored.